THE IDEAS OF ALGEBRA

The Mathematics of School Algebra

The introductory sessions involve the analysis of the mathematical foundations of the school mathematics curriculum from an advanced perspective (2 weeks)

- Klein, F. (1945). *Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra, Analysis*. New York: Dover Publications.
- Usiskin, Z., Peressini, A.L., Marchisotto, E., & Stanley, D. (2003). *Mathematics for High School Teachers- An Advanced Perspective*. Prentice Hall. Chapters 1-6.
- Wu, H. (Undated draft). *Introduction to school algebra.*

The Mathematics Curriculum Past and Present

The second unit involves the analysis and comparison of major curriculum recommendations as well as the examination of how recent recommendations are reflected in current mathematics programs (3 weeks).

Curriculum recommendations

- Smith, D.E. (1904, 1970) Algebra–What and why taught. In J. K. Bidwell & R. G. Clason (Eds.). *Readings in the history of mathematics education.* Reston, VA: National Council of Teachers of Mathematics.
- Mathematics Association of America (1923, 1970). The reorganization of mathematics in secondary education. In J. K. Bidwell & R. G. Clason (Eds.), *Readings in the history of mathematics education.* (pp. 442-459). Reston, VA: National Council of Teachers of Mathematics.
- Commission on Mathematics (1959). *Program for college preparatory mathematics.* New York: College Entrance Examination Board. (pp. 20-24)
- Cambridge Conference on School Mathematics (1963). *Goals for school mathematics.* (pp. 48-55, 61-66).
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council. (2001). *Adding it up.* Washington, DC: National Academies Press (Chapter 8).
- National Research Council. (1998): *The nature and role of algebra in the K-14 curriculum*. Washington, DC: National Academy Press.

Algebra Curricula

- Becker, J., & Jacob, B. (2000). The politics of California school mathematics: The Anti-reform of 1997-99. *Phi Delta Kappan*, 529 – 537.

- Blume, G. & Heckman, D. (1997). What do students know about algebra and functions? In P. Kenney & E. Silver (Eds.), *Results from the sixth mathematics assessment of the National Assessment of Educational Progress* (pp. 225-277). Reston, VA: NCTM.
- Donoghue, E. (2003). Algebra and geometry textbooks in twentieth century America. In G. M. Stanic & J. Kilpatrick (Eds.). *A history of school mathematics* (pp. 329-398). Reston, VA: NCTM.
- *Mathematics in Context (MiC)*
- *Connected Mathematics Project (CMP)*
- *Core Plus*
- *Discovering Algebra*
- Huntley, M.A., Rasmussen, C., Villarubi, R., Sangtong, J., & Fey, J. (2000). Effects of Standards-based mathematics education: A study for the CORE-Plus Mathematics Project algebra and functions strand. *Journal for Research in Mathematics Education, 31*(3), 328-361.
- Krebs, A. (2003). Middle grade students' algebraic understanding in a reform curriculum. *School Science and Mathematics, 103*(5), 233-243.
- Kilpatrick, J. (1997). Confronting reform. *American Mathematical Monthly, 104*, 955-62.
- Martin, T.S., Hunt, C.A., Lannin, J., Leonard Jr., W., Marshall, G.L., & Wares, A. (2001). How reform secondary mathematics textbooks stack up against NCTM's *Principles and standards*. *Mathematics Teacher, 94*(7), 540-546.
- Davydov, V. (1982). The psychological characteristics of the "prenumerical" period of mathematics instruction. In L. P. Steffe (Ed.) *Soviet studies in the psychology of learning and teaching mathematics* (Vol. 7, pp. 109-206). Chicago: The University of Chicago Press.

GENERAL RESEARCH ISSUES (1week)

- Rand Mathematics Study Panel (2003). *Mathematical proficiency for all students: Toward a strategic and development program in mathematics education*. Washington, DC: Rand.
- Kieran, C. & Wagner, S. (1989). The research agenda conference on algebra: Background and issues; An agenda for research on the learning and teaching of algebra. In C. Kieran & S. Wagner (Eds), *Research issues in the learning and teaching of algebra*. (pp. 1-10, 220-237). Reston, VA: NCTM.

RESEARCH ON STUDENTS' LEARNING OF ALGEBRA

Much of the research in algebra has focused on student learning of algebra. Up until the 1990s, much of the work documented students' misconceptions and the difficulties they had in learning algebra. More recently research has been studying what students can do, showing that younger children are capable of learning fundamental algebra ideas earlier than previously thought. Our study of the research is separated in four parts: (a) research on the development of algebraic reasoning in the early grades; (b) research on the development of foundational ideas in algebra such as proportional reasoning, ratio and rate, pattern recognition and generalization; and (c) research related to the learning of algebra concepts; and (d) cross cutting issues of instruction of algebra.

Early Algebra (Two weeks)

Dealing with Misconceptions about the Use of the Equal Sign

- Baroody, A. & Ginsburg, H. (1983). The effects of instruction on children's understanding of the "equals" sign. *Elementary School Journal, 84*(2), 199-212.
- Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equal sign. *Mathematics Teaching, 92,* 13–15.
- Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating algebra and arithmetic in elementary school*. Portsmouth, NH: Heinemann. (Chapter 2).
- Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics, 6,* 232–236.
- Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12,* 317–326.
- Matz, M. (1982). Towards a process model for school algebra errors. In D. Sleeman & J. S. Brown (Eds.), *Intelligent tutoring systems* (pp. 25–50). New York: Academic Press.
- McNeil, N., Grandau, L., Knuth, E., Alibali, M., Stephens, A., Hattikudur, S., & Krill, D. (Under review). Middle school students' understanding of the equal sign: The books they read can't help. *Cognition & Instruction*.
- Saenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics, 35,* 153–187.
- Seo, K-H. & Ginsburg, H. P. (2003). "You've got to carefully read the math sentence…": Classroom context and children's interpretations of the equals sign. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills*. Mahwah, NJ: Erlbaum.

Reasoning Algebraically: Learning Arithmetic as a Foundation for Learning Algebra

- Bastable, V. & Schifter, D. (In press): Classroom stories: Examples of elementary students engaged in early algebra. - In: J. Kaput (Ed.), *Employing children's natural powers to build algebraic reasoning in the context of elementary mathematics*.
- Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating algebra and arithmetic in elementary school*. Portsmouth, NH: Heinemann. (Chapters 1, 3-10).
- Carpenter, T. P., Levi, L., Berman, P., & Pligge, M. (2005). Developing algebraic reasoning in the elementary school. In T. A. Romberg, T. P. Carpenter, & F. Dremock (Eds.), *Understanding mathematics and science matters* (pp. 81-98). Mahwah, NJ: Erlbaum.
- Carraher, D., Brizuela, B., & Earnest, D. (2001). The reification of additive differences in early algebra: Viva la difference! In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), P*roceedings of the 12th ICMI study conference: The future of teaching and learning of algebra.* (pp. 163-170). Melbourne, Australia: University of Melbourne.
- Davis, R. B. (1984). ICMI-5 report: Algebraic thinking in early grades. *The Journal of Mathematical Behavior, 4,* 195-208.
- Verschaffel, L., Greer, B., & DeCorte, E. (2000) Making sense of word problems. Lisse, Netherlands: Swets & Zeitlinger.

- Fujii, T. & Stephens, M. (2001). Fostering an understanding of algebraic generalization through numerical expressions: The Role of quasi-variables. In H. Chick, K. Stacey., J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The Future of the teaching and learning of algebra* (pp. 258 - 264). Melbourne, Australia: The University of Melbourne.
- Gallardo, A. (2002), The extension of the natural number domain to the integers in the transition from arithmetic to algebra, *Educational Studies in Mathematics, 49*, 171-192.
- Kaput, J. (1998). Transforming algebra from an engine of inequity to an engine of mathematical power by "algebrafying" the K–12 curriculum. In National Council of Teachers of Mathematics (Eds.), *The nature and role of algebra in the K–14 curriculum.* Washington, DC: National Academy Press.
- Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 133–155). Mahwah, NJ: Erlbaum.
- Kaput, J. J. & Blanton, M. L. (2005). A teacher-centered approach to algebrafying elementary mathematics. In T. A. Romberg, T. P. Carpenter, & F. Dremock (Eds). *Understanding mathematics and science matters* (pp. 99-126). Mahwah, NJ: Erlbaum.
- Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33–56*).* Reston, VA: National Council of Teachers of Mathematics.
- Schifter, D. (1999). Reasoning about operations: Early algebraic thinking in Grades K–6. In L. V. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in Grades K–12* (pp. 62–81). Reston, VA: National Council of Teachers of Mathematics.
- van Reeuwijk, M. (in press). Early school algebra: A Dutch perspective. In J. Kaput, D. Carraher & M. Blanton (Eds.), *Employing children's natural powers to build algebraic reasoning in the content of elementary mathematics.*

Development of Foundational Ideas (Two weeks)

Pattern Generalization in Algebra

- Curcio, F., Nemirovsky, R., Perez, R., & Yaloz, S. (1997). Exploring patterns in nonroutine problems. *Mathematics Teaching in the Middle School, 2*(4), 262 – 269.
- Ellis, A.B. (in press). A taxonomy for categorizing generalizations: generalizing actions and reflection generalizations.
- Garcia-Cruz, J., & Martinon, A. (1997). Actions and invariant in schemata in linear generalizing problems. In E. Pehkonen (Ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 289-296). Lahti, Finland: PME.
- Lannin, J.K. (2003). Developing algebraic reasoning through generalization. *Mathematics Teaching in the Middle School, 8*(7), 342.
- Lee, L. (1996). An initiation into algebraic culture through generalization activities. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra* (pp. 87 – 106). Dordrecht, the Netherlands: Kluwer.
- Petocz, P., & Petocz, D. (1997). Pattern and proof: The art of mathematical thinking. *The Australian Mathematics Teacher, 53*(3), 12 – 17.

- Radford, L. (1996). Some reflections on teaching algebra through generalization. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra* (pp. 107 - 111). Dordrecht, The Netherlands: Kluwer.

Development of Linearity / Proportional Reasoning

- Cramer, K. & Post, T. (1993, May). Connecting research to teaching proportional reasoning. *Mathematics Teacher,* 86(5), 404-407.
- Karplus, R., Pulos, S., & Stage, E. K. (1983). Proportional reasoning of early adolescents. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 45-90). Orlando, FL: Academic Press.
- Lamon, S. J. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In G. Harel, & J Confrey (Eds.), The development of multiplicative reasoning in the learning of mathematics (pp. 89-120). Albany, NY: State University of New York Press.
- Lamon, S.J. (1995). Ratio and proportion: Elementary didactical phenomenology. In J.T. Sowder & B.P. Schappelle (Eds.). *Providing a foundation for teaching mathematics in the middle grades* (pp. 167 – 198). Albany, NY: State University of New York Press.
- Lesh, R., Post, T., & Behr, M. (1988) Proportional reasoning. In J. Hiebert & M. Behr (Eds) Number concepts and operations in the middle grades (pp. 93-118). National Council of Teachers of Mathematics: Reston, VA.
- Lobato, J., & Thanheiser, E. (2002). Developing understanding of ratio and measure as a foundation for slope. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 162 – 175). Reston, VA: National Council of Teachers of Mathematics.
- Miller, J., & Fey, J. (2000). Proportional reasoning. *Mathematics Teaching in the Middle School, 5*(5), 310-313.
- Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics, 16,* 181-204.

Learning of Fundamental Algebra Ideas (Three weeks)

Overview: Cognitive / Theoretical Issue

- Chazan, D. & Yerushalmy, M. (2002). On appreciating the cognitive complexity of school algebra: Research on algebra learning and directions of curricular change. In Kilpatrick, J., Schifter, D. & G. Martin (Eds.), *A Research Companion to the Principles and Standards for School Mathematics,* Reston: NCTM.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 390 – 419). New York: Macmillan Publishing Company.
- English, L., & Warren, E. (1995). General reasoning processes and elementary algebraic understanding: Implications for instruction. *Focus on Learning Problems in Mathematics, 17*(4), 1 – 19.
- Herscovics, N. & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27,* 59-78.
- Herscovics (1989) Cognitive obstacles encountered in the learning of algebra. In S.

Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 60-86). Reston, VA: National Council of Teachers of Mathematics.
- Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 121-142. Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra* (pp. 65 – 86). Dordrecht, the Netherlands: Kluwer.
- Nickson, M. (2000). *Teaching and learning mathematics: A teacher's guide to recent research and its application*. New York: Cassell. [Chapter 4]
- Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *The Journal of Mathematical Behavior, 14,* 15-39.
- Sfard, A. & Linchevski, L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics, 26,* 191-228.

Misconceptions

- Booth, L.R. (1988). Children's difficulties in beginning algebra. In A. Coxford (Ed.), *Ideas of Algebra: K-12* (pp. 20-32). Reston, VA: NCTM.
- Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13,* 16–30.
- Kaput, J., & Clement, J. (1979). Letter to the editor. *Journal of Mathematical Behavior, 2,* 208.
- Kaput, J.J., Sims-Knight, J.E., & Clement, J. (1985). Behavioral objections: A response to Wollman. *Journal for Research in Mathematics Education, 16*(1), 56-63.
- Philipp, R.A. (1992). A study of algebraic variables: Beyond the student-professor problem. *The Journal of Mathematical Behavior, 11*(2), 161-176.
- Rosnick, P., & Clement, J. (1980). Learning without understanding: The effect of tutoring strategies on algebra misconceptions. *Journal of Mathematical Behavior, 3*, 3–27.
- Wollman, W. (1983). Determining the sources of error in a translation from sentence to equation. *Journal for Research in Mathematics Education, 14*(3), 169-181.
- Wollman, W. (1985). A reply to Kaput, Sims-Knight, and Clement. *Journal for Research in Mathematics Education, 16*(1), 63-66.

Conceptions of Equations

- Davis, R. B. (1975). Cognitive processes involved in solving simple algebraic equations. Journal of Children's Mathematical Behavior, 1 (3), 7-35.
- Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics,* 12, 317-326.
- Knuth, E., Stephens, A., McNeil, N., & Alibali, M. (In press). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education.*

- Nathan, M. & Koedinger, K. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education, 31*(2), 168-190.
- Steinberg, R.; Sleeman, D.; & Ktorza, D. (1990): Algebra students' knowledge of equivalence of equations. *Journal for Research in Mathematics Education, 22*(2), 112-121.
- Swafford, J. & Langrall, C. (2000). Grade 6 students' preinstructional use of equations to describe and represent problem situations. *Journal for Research in Mathematics Education, 31*(1), 89-112.

Conceptions of Functions

- Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992) Development of the process conception of function. *Educational Studies in Mathematics, 23*, 247-285.
- Carlson, M.P. (1997). Obstacles for college algebra students in understanding functions: What do high-performing students really know? *The AMATYC Review, 19*(1):48-59.
- Clement, L. (2001). What do your students really know about functions? *Mathematics Teacher, 94*, 745-748.
- Eisenberg, T. (1992). On the development of a sense for functions. In G. Harel & E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 153-174). Washington, D.C.: Mathematical Association of America.
- Hirsch, C. (1997). Functions and relations: A unifying theme for school algebra in grades 9 – 12. In *The Nature and role of algebra in the K-14 curriculum: Proceedings of a national symposium* (pp. 61 – 62). Washington, DC: National Council of Teachers of Mathematics.
- Hollar, J., & Norwood, K. (1999). The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function. *Journal for Research in Mathematics Education, 30*(2), 220-226.
- Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993) Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema, & T. P. Carpenter Eds.), *Integrating research on the graphical representation of function* (pp. 69-100). Hillsdale, NJ: Lawrence Erlbaum.
- O'Callaghan, B. R. (1998). Computer-intensive algebra and students' conceptual knowledge of functions. *Journal for Research in Mathematics Education, 29*(1), 21-40.
- Zaslavsky, O. (1997). Conceptual obstacles in the learning of quadratic functions. *FOCUS on Learning Problems in Mathematics, 19*(1), 20-44.

Conceptions of Literal Symbols

- Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in School, 7*(4), 23-26.
- MacGregor, M. & Stacey, K. (1997) Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics, 33*, 1-19.
- Philipp, R.A. (1992). The many uses of algebraic variables. *The Mathematics Teacher, 85*(7), 557-561.

- Stacey, K. & MacGregor, M. (1997). Ideas about symbolism that students bring to algebra. *Mathematics Teacher, 90*(3).
- Stephens, A. C. (In press). Developing students' understandings of variable. *Mathematics Teaching in the Middle School*.
- Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A.F. Coxford (Ed.), *The Ideas of Algebra, K-12* (pp. 8-19). Reston, VA: NCTM.

Conceptions of Graphs

- Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. For the *Learning of Mathematics*, 2 (1), 34-42.
- Clement (1989) The concept of variation and misconception in Cartesian graphing. *Focus on Learning Problems in Mathematics, 11*(1-2), 77-87.
- Freil, S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education, 32*(2), 124-158.
- Goldenberg, P. (1988). Mathematics, metaphors, and human factors: Mathematical, technical, and pedagogical challenges in the educational use of graphical representation of functions. *Journal of Mathematical Behavior* (7), 135-173.
- Kieran, C. (1993). Functions, graphing, and technology: Integrating research on learning and instruction. In T. A. Romberg, E. Fennema, & T. P. Carpenter Eds.), *Integrating research on the graphical representation of function* (189-238). Hillsdale, NJ: Lawrence Erlbaum.
- Knuth, E. (2000). Student understanding of the Cartesian Connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500-508.
- Larkin, J., & Simon, H. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65-99.
- Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64.

Instruction (2 weeks)

- Chazan, D. (2000). *Beyond formulas in mathematics teaching: Dynamics of the high school algebra classroom.* New York: Teachers College.
- Even, R. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 95-116.
- Haimes, D. (1996). The implementation of a "function" approach to introductory algebra: A case study of teacher cognitions, teacher actions, and the intended curriculum. *Journal for Research in Mathematics Education, 27*(5), 582-602.
- Hall, R., & Rubin, A. (1998). There's five little notches in here: Dilemmas in teaching and learning the conventional structure of rate. In J. Greeno & S. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 189 – 236). Mahwah, NJ: Lawrence Erlbaum Associates.

- Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education,* 2, 227-252.
- Lobato, J., Ellis, A.B., & Muñoz, R. (2003). How "focusing phenomena" in the instructional environment afford students' generalizations. *Mathematical Thinking and Learning, 5*(3), 1-36.
- Malloy, C. (1997). Mathematics projects promote students' algebraic thinking. *Mathematics Teaching in the Middle School, 2*(4), 282 – 288.
- Nathan, M., & Koedinger, K. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction, 18*(2), 209–237.
- Wagner, S. & Kieran, C. (Eds.) (1989). *Research Agenda for Mathematics Education Research Issues in the Learning and Teaching of Algebra.* Lawrence Erlbaum Associates.

Use of Technology

- Drijvers, P. (2003). *Learning algebra in a computer algebra environment: design research on the understanding of the concept of parameter*. (Chapter 5).
- Edwards, M.T. (2004). Novice algebra students may be ready for CAS – but are CAS tools ready for novice algebra students?' *International Journal of Computer Algebra in Mathematics Education, 10*(4),
- Gage, J.A. (2002). Using the graphic calculator to form a learning environment for the early teaching of algebra. *The International Journal of Computer Algebra in Mathematics Education, 9*(1), 3-27.
- Kieran, C. (1993). Functions, graphing and technology: Integrating research on learning and technology. In T. A. Romberg, T. P. Carpenter, & E. Fennema (Eds.). *Integrating research on the graphical representation of functions.* (pp. 189-238). Mahwah, NJ: Earlbaum.
- Lobato, J., & Ellis, A.B. (2002). Focusing effects of technology: Implications for teacher education. *Journal of Technology and Teacher Education, 10*(2), 297 - 314.
- Merriweather, M., & Tharp, M. (1999). The effect of instruction with graphing calculators on how general mathematics students naturalistically solve algebraic problems. *Journal of Computers in Mathematics and Science Teaching 18*(1), 7-22.
- Parnafes, O., & Disessa, A. (2004). Relations between types of reasoning and computational representations. *International Journal of Computers for Mathematical Learning, 9*(3), 251-280.
- Pea. R. Cognitive technologies for mathematics education. In Schoenfeld, A. H. (Ed.) (1987). *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, NJ: Erlbaum.
- Schoenfeld, A. H. Mathematics, technology, and higher order thinking. In R. S. Nickerson and P. D. Zodhiates (Eds.), *Technology in education: Looking toward 2020,* pp. 67-96. Hillsdale, NJ: Erlbaum, 1988.
- Yerushalmy, M. & Chazan D. (2002). Technologically supported curricular change, teacher knowledge, and student learning: The case of graphing and solving in school algebra. To appear in L. English et al. (Eds.*) Handbook of international research in mathematics education* (pp. 725-755). Hillsdale, NJ: Erlbaum.

Use of Manipulatives

- Leitze, A. & Kitts, N. (2000). Using homemade algebra tiles to develop algebra and prealgebra concepts. *Mathematics Teacher, 93*(6), 462-466.
- Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 121-142.
- Peck, D.M., & Jencks, S.M. (1988). Reality, arithmetic, and algebra. *Journal of Mathematical Behavior, 7*, 85-91.
- Sharp, J.M. (1995). Results of using algebra tiles as meaningful representations of algebra concepts. *Paper presented at the Annual Meeting of the Mid-Western Educational Research Association.*
- Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E. (2000). Using algebra tiles to multiply monomials and binomials: The case of Monique Butler. Chapter 8 in *Implementing Standards-Based Mathematics Instruction* (pp. 96 – 103). New York: Teachers College Press.

EQUITY AND THE TEACHING AND LEARNING OF ALGEBRA (One week)

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights.* Boston, MA: Beacon Press.

GENERAL REFERENCES

- Bednarz, N., Kieran, C., & Lee, L. (1996). *Approaches to algebra: Perspectives for research and teaching*. Dordrecht: Kluwer Academic Press.
- Chick, H., Stacey, K., Vincent, J., & Vincent, J. (Eds.) (2001). *Proceedings of the 12th ICMI study conference: The future of teaching and learning of algebra.* Melbourne, Australia: University of Melbourne.
- Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6–10*. Portsmouth, NH: Heinemann.
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- National Council of Teachers of Mathematics. (1998). *The nature and role of algebra in the K–14 curriculum.* Washington, DC: National Academy Press.
- Wagner, S. & Kieran, C. (1989). *Research issues in the learning and teaching of algebra*. Reston, VA: National Council of Teachers of Mathematics.