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Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning

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In order to develop students' capacities to "do mathematics," classrooms must become environments in which students are able to engage actively in rich, worthwhile mathematical activity. This paper focuses on examining and illustrating how classroom-based factors can shape students' engagement with mathematical tasks that were set up to encourage high-level mathematical thinking and reasoning. The findings suggest that when students' engagement is successfully maintained at a high level, a large number of support factors are present. A decline in the level of students' engagement happens in different ways and for a variety of reasons. Four qualitative portraits provide concrete illustrations of the ways in which students' engagement in high-level cognitive processes was found to continue or decline during classroom work on tasks.

During the past decade, much discussion and concern have been focused on limitations in students' conceptual understanding as well as on their thinking, reasoning, and problem-solving skills in mathematics (Hiebert & Carpenter, 1992; Lindquist & Kouba, 1989; National Research Council, 1989). In response to these concerns, the National Council of Teachers of Mathematics (NCTM) has published proposed reforms of curriculum, evaluation, and teaching practices commonly found in primary and secondary school mathematics classrooms (NCTM, 1989, 1991, 1995). Among the underlying goals of these reform efforts are to enhance students' understanding of mathematics and to help them become better mathematical doers and thinkers.

What does it mean to be a mathematical doer and thinker? Answers to this question depend on one's view of the nature of mathematics. A view of mathematics that has gained increasing acceptance over past years is one based on a dynamic and exploratory stance toward the discipline (Romberg, 1994). This dynamic stance toward mathematics requires one to focus on the active, generative processes engaged in by doers and users of mathematics (Schoenfeld, 1992), rather than view mathematics as a static, structured system of facts, procedures, and concepts. Such active mathematical

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processes involve the use of mathematical tools systematically to explore patterns, frame problems, and justify reasoning processes (Burton, 1984; National Research Council, 1989; Romberg, 1992; Schoenfeld, 1992, 1994).

This more dynamic notion of mathematical activity has implications for ideas about what students need to learn and the kinds of activities in which students and teachers should engage during classroom interactions. Students' learning is seen as the process of acquiring a "mathematical disposition" or a "mathematical point of view" (Schoenfeld, 1992, 1994), as well as acquiring mathematical knowledge and tools for working with and constructing knowledge. Having a mathematical disposition is characterized by such activities as looking for and exploring patterns to understand mathematical structures and underlying relationships; using available resources effectively and appropriately to formulate and solve problems; making sense of mathematical ideas, thinking and reasoning in flexible ways: conjecturing, generalizing, justifying, and communicating one's mathematical ideas; and deciding on whether mathematical results are reasonable (Schoenfeld, 1992). These activities have much in common with the active reasoning processes that Resnick (1987) and others have proposed as characteristics of high-level thinking in a variety of academic domains. If students are to develop these capacities, then classrooms must become environments in which they have frequent opportunities to engage in dynamic mathematical activity that is grounded in rich, worthwhile mathematical tasks (NCTM, 1991; Schoenfeld, 1994).

Importance of Mathematical Instructional Tasks

Mathematical tasks are central to students' learning because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). The tasks in which students engage provide the contexts in which they learn to think about subject matter, and different tasks may place differing cognitive demands on students (Doyle, 1983; Marx & Walsh, 1988; Hiebert & Wearne, 1993). Thus, the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged. Students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage (Schoenfeld, 1992, 1994).

How feasible is it to engage students consistently and successfully in high-level tasks for "doing mathematics" in the classroom? Academic task researchers (Doyle 1983, 1986, 1988) have noted that high-level tasks are often complex and longer in duration than more routine classroom activities and are thus more susceptible to various factors that could cause a decline in students' engagement to less demanding thought processes. A previous study of mathematical tasks in reform classrooms at the middle-school level (Stein, Grover, & Henningsen, 1996) obtained results that further substantiated the difficulty of maintaining high levels of students' cognitive processing throughout task implementation. These findings suggest that although attention to the nature of mathematical instructional tasks is important, attention to the classroom processes surrounding mathematical tasks is equally needed. Although much has been written about types of mathematical tasks that afford students opportunities to do mathematics, there have been fewer investigations of the kinds of instructional environments required to support the implementation of tasks for “doing mathematics.” This paper addresses that gap by focusing on classroom-based factors that influence the ways in which students engage with cognitively demanding mathematics tasks in real classroom settings.

Difficulties Associated with Implementing High-Level Tasks

Tasks that are set up to engage students in cognitively demanding activities often evolve into less demanding forms of cognitive activity (Doyle 1983, 1986, 1988). Engaging in high-level reasoning and problem solving involves more ambiguity and higher levels of personal risk for students than do more routine activities. Such engagement can evoke in students a desire for a reduction in task complexity that, in turn, can lead them to pressure teachers to further specify the procedures for completing the task or to relax accountability requirements (Doyle, 1983). There may also be a tendency for classroom-based work on tasks to drift away from a focus on meaning and understanding toward an emphasis on accuracy and speed (Doyle, 1988). Another factor underlying unsuccessful task implementation is a lack of alignment between tasks and students’ prior knowledge, interests, and motivation (Bennett & Desforges, 1988). Such mismatches may cause students to fail to engage with the task in ways that will maintain a high level of cognitive activity.

In general, a complex array of factors is involved in orchestrating classroom activity and balancing classroom management needs with academic demands. Factors can be rooted in the way classroom norms are set up, in the motivation and learning dispositions of students, and in the general classroom management practices of teachers. These factors include the manner in which order is established in the classroom, the physical organization of the environment, the amount of time allotted for various activities, the manner in which transition periods between tasks are handled, the establishment of accountability structures, and the ways in which discipline interventions are handled (Doyle, 1986). However, tasks that begin as cognitively demanding ones do not always decline, and it is equally important to understand ways in which high-level cognitive demands can be *maintained* as the tasks are implemented in the classroom.

Ways of Supporting Implementation of High-Level Tasks

When students “do mathematics” in the classroom, the activity has most likely not occurred in a vacuum. Factors that contribute to the decline of high-level demands, when considered in the reverse, can point to ways of maintaining high-level demands. For example, Doyle (1988) argued that teachers should be especially attentive to the extent to which meaning is emphasized and the extent to which students are explicitly expected to demonstrate understanding of the mathematics underlying the activities in which they are engaged. Such an emphasis can be maintained if explicit connections between the mathematical ideas and the activities in which students engage are frequently drawn. Connections with what students already know and understand also play an important role in engaging students in high-level thought processes (Hiebert & Carpenter, 1992). Some researchers have pointed out that if cognitively demanding tasks *are* appropriate with respect to students' levels and kinds of prior knowledge, students' cognitive processing during task implementation stands a better chance of remaining at a high level (e.g., Bennett & Desforges, 1988). Also, structuring classroom activity so that appropriate amounts of time are devoted to tasks is important (Doyle, 1986).

In a review of research on classroom instruction for high-level understanding, Anderson (1989) supported these ideas as well as some others. Anderson noted the importance of the Vygotskian notion of *scaffolding* in helping students to understand and make connections among important ideas. Scaffolding occurs when a student cannot work through a task on his or her own, and a teacher or more capable peer provides assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task. Also, teachers can support high-level thinking processes in students by explicitly modeling (or by having a student model) such processes and thinking strategies (Anderson, 1989). Finally, it is important to encourage students to engage in self-monitoring or self-questioning as they progress through a task (Anderson, 1989; Schoenfeld, 1983; Silver, Branca, & Adams, 1980). Self-monitoring can increase students' feelings of competence and control and, in turn, their motivation to remain engaged with a task at a high level.

These findings suggest that the mere presence of high-level mathematical tasks in the classroom will not automatically result in students' engagement in doing mathematics. Without engaging in such active processes during classroom instruction, students cannot be expected to develop the capacity to think, reason, and problem solve in mathematically appropriate and powerful ways. Clearly, the ambient classroom environment must actively support successful engagement of students in high-level thinking and reasoning.

This paper investigates the classroom factors that either hinder or support students'engagement in high-level mathematical thinking and reasoning for doing mathematics. The context for the present investigation consists of mathematics classrooms that are participating in the QUASAR project[1], a national educational reform project aimed at fostering and studying the development and implementation of enhanced mathematics instructional programs for students attending middle schools in economically disadvantaged communities (Silver & Stein, 1996). The project is based on the premise that prior failures of poor and minority students are due to a lack of opportunities to participate in meaningful and challenging learning experiences rather than to a lack of abilities or potential. Since the fall of 1990, groups of mathematics teachers at six geographically dispersed and ethnically diverse urban middle schools have been working, in collaboration with resource partners from nearby universities, to enhance their own local instruction and professional

[1]QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) is based at the Learning Research and Development Center at the University of Pittsburgh and is directed by Edward A. Silver.

development programs in order to provide their students with good mathematics instruction aimed at fostering thinking, reasoning, and problem solving. Although QUASAR teachers have received a broad array of staff development since the inception of the project, the teachers' educational and professional backgrounds were typical of most middle school mathematics teachers (QUASAR Documentation Team, 1993). Compared to the national profile, QUASAR teachers are more ethnically diverse.

CONCEPTUAL FRAMEWORK

The present study is guided by a conceptual framework based on the construct of a mathematical instructional task (see Stein et al., 1996, for a more detailed overview of the content and design of the framework). The framework, shown in Figure 1, defines a mathematical task as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill.

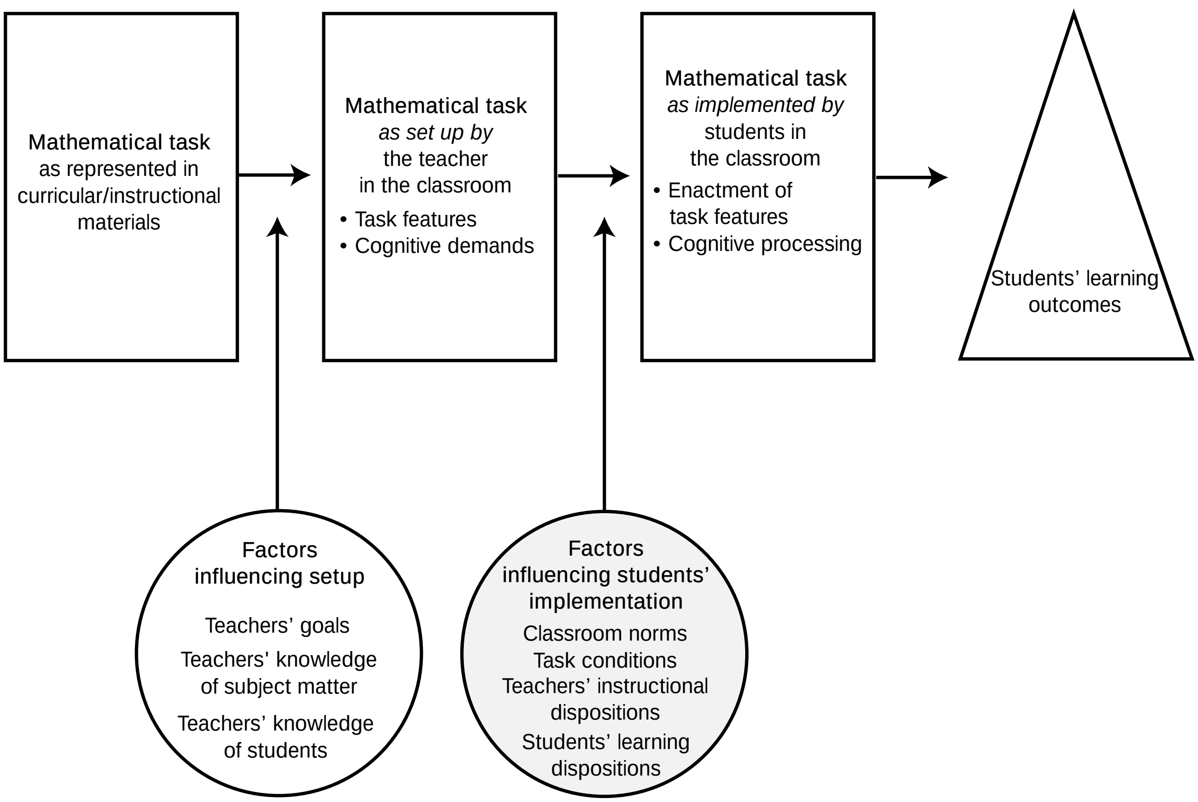

Figure 1. Relationships among various task-related variables and students' learning outcomes. The shaded portion represents the area primarily under investigation.

In this framework mathematical tasks pass through three phases (represented by the rectangular boxes in Figure 1): as written by curriculum developers, as set up by the teacher in the classroom, and as implemented by students during the lesson. The framework further specifies two dimensions of mathematical tasks. The first dimension is *task features*. Task features refer to aspects of tasks that mathematics educators have identified as important considerations for the development of mathematical understanding, reasoning, and sense making. These features include multiple solution strategies, multiple representations, and mathematical communication. During the set-up phase, these features refer to the extent to which the task as announced by the teacher encourages students to use more than one strategy, to use multiple representations, and to supply explanations and justifications. During the implementation phase, these features refer to the extent to which students use the features.

The second dimension, *cognitive demands*, refers to the kind of thinking processes entailed in solving the task as announced by the teacher (during the set-up phase) and the thinking processes in which students engage (during the implementation phase). These thinking processes can range from memorization to the use of procedures and algorithms (with or without attention to concepts, understanding, or meaning) to complex thinking and reasoning strategies that would be typical of “doing mathematics” (e.g., conjecturing, justifying, or interpreting). The present investigation focused on this second dimension of cognitive demands and the classroom-based factors that influenced them as tasks passed from the set-up to the implementation phase.

According to the framework, the features and cognitive demands of tasks can be transformed between any two successive phases. For example, a task could be set up to require high-level cognitive activity by students, but during the implementation phase it could be transformed in such a way that students’ thinking focuses only on procedures, with no conceptual connections. The shaded circle in Figure 1 represents the classroom-based factors that influence the ways in which students’ thinking unfolds during the task-implementation phase. These factors include classroom norms, task conditions, and teachers’ and students’ dispositions. *Classroom norms* refers to the established expectations regarding how academic work gets done, by whom, and with what degree of quality and accountability. *Task conditions* refers to attributes of tasks as they relate to a particular set of students (e.g., the extent to which tasks build on students’ prior knowledge and the appropriateness of the amount of time that is provided for students to complete tasks). *Teacher and student dispositions* refers to relatively enduring features of pedagogical and learning behaviors that tend to influence how teachers and students approach classroom events. Examples include the extent to which a teacher is willing to let a student struggle with a difficult problem, the kinds of assistance that teachers typically provide students who are having difficulties, and the extent to which students are willing to persevere in their struggle to solve difficult problems. Through these classroom, task, and teacher and student factors, tasks can be shaped by the ambient classroom culture.

PURPOSE OF THE STUDY

The purpose of the present study is to identify, examine, and illustrate the ways in which classroom-based factors shape students’ engagement with high-level mathematical tasks. Previous work (Stein et al., 1996) has identified various patterns of student engagement with tasks that were set up to encourage doing mathematics.

In some cases, students engaged actively in high-level cognitive processes characteristic of doing mathematics. In other cases they did not. In those cases in which students' engagement with the tasks did not exemplify doing mathematics, three characteristic types of student engagement were noted: cognitive activity that focused on the mechanical use of procedures (with no connection to underlying meaning), unsystematic exploration, and activity with no mathematical focus.

This study identifies and describes both those profiles of factors that were associated with maintaining high levels of cognitive demand and those with each of the characteristic patterns of decline. In addition, the study includes classroom-based illustrations of the maintenance and decline patterns noted above and the factor profiles associated with them.

METHODOLOGY

Data for the present study were drawn from an earlier investigation in which instruction in a representative sample of QUASAR classrooms was examined (Stein, et al., 1996). This earlier study focused on the nature of mathematical tasks as vehicles for building student capacity for mathematical thinking and reasoning.

Prior Investigation

Data sources. Trained and knowledgeable observers[2] wrote narrative summaries of classroom observations. These summaries formed the primary basis of the data used in the initial study. Each school year from the fall of 1990 to the spring of 1993, three 3-day observation sessions (fall, winter, and spring) were conducted in three representative teachers' mathematics classrooms at each of four project sites. The classroom-based illustrations described in this study were drawn from the classrooms of four project teachers: Mr. Hernandez, Ms. Capra, Ms. Hoffman, and Mr. Kingsley. The observer took detailed field notes on the mathematics instruction and students' reactions to the instruction; a camera operator simultaneously videotaped the lesson. Following the observations, the observer used both the videotaped lesson and his or her field notes to complete the project's Classroom Observation Instrument (COI).[3] As part of that instrument, the observer provided descriptions and sketches of the physical setting of the room, a chronology of instructional events, and

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[2]Observers selected had strong backgrounds in mathematics education, educational psychology, or a related field; a demonstrated competence in analyzing instruction; prior experience observing classrooms; and an understanding of ethnic or multicultural issues at the various sites. The observers underwent extensive training and a sample of their write-ups were reviewed and feedback was provided.

[3]The initial draft of this instrument drew from two main sources: NCTM's *Professional Standards for Teaching Mathematics:* Working Draft (1989), and a classroom observation system used for the State of California study of elementary school mathematics (Cohen, D. K., Peterson, P. L., Wilson, S., Ball, D., Putnam, R., Prawat, R., Heaton, R., Remillard, J., & Wiemers, M. (1990). Effects of state level reform of elementary school mathematics curriculum on classroom practice [Elementary Subject Center Series No. 25], East Lansing, Michigan State University, Center for the Learning and Teaching of Elementary Subjects and the National Center for Research on Teacher Education). The instrument has been pilot tested in several middle school mathematics classrooms and has undergone several rounds of critique and revisions.

responses to questions associated with five themes: mathematical tasks, classroom discourse, the intellectual environment, management and assessment practices, and group work (if it occurred).

In the COI, a mathematical task is defined as a segment of classroom work that is devoted to learning about a particular mathematical idea. The observers were instructed to segment the instructional time of each observed lesson according to the main mathematical tasks with which students were engaged. The artifacts associated with these tasks were appended to the write-up. The two tasks that occupied the largest percentages of class time were designated as Task A and Task B. In the mathematical tasks section of the COI, the observer described in detail the nature of these two tasks: their mathematical content, the learning goals of the teacher for each task, and the behaviors of the students as they engaged in these tasks. The observer also described the extent to which each task focused students' attention on procedural steps with or without connections to underlying concepts and on doing mathematics (e.g., framing problems, making conjectures, justifying, and explaining). In the remaining three sections of the COI, the observer wrote about all activities that occurred during the classroom lesson (not limited to Task A and Task B), referring specifically to the two main tasks when appropriate.

Coding procedures. For the initial study, a sample of 144 COIs was selected for coding with the goal of gaining a representative picture of instruction across the four project sites and the first 3 project years. Only Task A of each observation was coded, although the entire narrative summary for the classroom observation was reviewed and considered in making coding decisions.[4] The COIs were coded using a system based on the conceptual framework shown in Figure 1. The coding system was initially developed on the basis of a review of the literature on academic tasks (Bennett & Desforges, 1988; Doyle, 1983, 1988; Marx & Walsh, 1988) and the cognitive psychology of instruction (Anderson, 1989), the literature on mathematical thinking and problem solving (Grouws, 1992; Silver, 1985), and mathematics reform documents (NCTM, 1989, 1991), as well as on knowledge of the project sites and their goals.

Nineteen coding decisions, organized into four main categories, were made for each task. *Descriptive* codes included the number of minutes and percentage of class time devoted to the task, the type(s) of resource(s) that served as the basis for the task, the mathematical topic of the task (conventional middle-school topic, reform-inspired topic, focus on mathematical processes more than a particular topic), the context of the task, and whether or not the task was set up as a collaborative venture among students.

Set-up codes were assigned on the basis of a review of the task materials (provided as appendices to the COI write-up) and the task as specified by the teachers, both during their initial announcements of what students were to do and during the

[4]Videotapes of observations and additional artifacts from an observation were used as supplemental data sources on a small number of the tasks (8%). These sources were consulted when a coder determined that the written description of the observation did not provide sufficient information on which to base a decision.

task at any subsequent points at which the teachers unilaterally provided additional specifications to guide students' approaches to the task. During the set-up phase, codes were assigned for task features and for the cognitive demands of the task. (Only cognitive-demand codes were used in the present study.) The cognitive demands were classified into one of the following: memorization; the use of formulas, algorithms, or procedures *without connection* to concepts, understanding, or meaning; the use of formulas, algorithms, or procedures *with connection* to concepts, understanding, or meaning; and cognitive activity that can be characterized as "doing mathematics," including complex mathematical thinking and reasoning activities such as making and testing conjectures, framing problems, and looking for patterns.

Implementation codes also were made for task features and cognitive demands. These codes were assigned with reference to the ways in which students went about working on the task. When coding the cognitive demands of the task as implemented, coders were asked to make judgments about the kinds of cognitive processes in which the majority of the students appeared to be engaged. During this phase, coders independently recognized the need for a new code to describe a frequently observed manner of implementing doing-mathematics tasks, one in which students explored around the edges of significant mathematical ideas but failed to make systematic and sustained progress in developing mathematical strategies or understandings. In the present article, this new code is called *unsystematic exploration*.

The final category of codes included judgments about *factors* associated with task implementation. For high-level tasks that remained so during implementation, coders were instructed to identify as many as applied from a list of possible factors that could assist with the maintenance of tasks at high levels (e.g., the modeling of high-level performance by teachers or capable students; sustained press for justification, explanations, or meaning through teacher questioning, comments, and feedback; scaffolding [teachers or more capable students simplifying the task so that it could be solved while maintaining task complexity]; or the selection of tasks that build on students' prior knowledge). In the earlier study, *high level* was used to describe tasks that involved doing mathematics or the use of formulas, algorithms, or procedures with connection to concepts, understanding, or meaning. For high-level tasks that declined, coders identified the reasons for the decline from a list of possibilities that included the routinization of problematic aspects of the tasks (students press teacher to reduce task ambiguity or complexity by specifying explicit procedures or the teacher "takes over" difficult pieces of the task); the shifting of emphasis from meaning, concepts, or understanding to the accuracy and completeness of answers; the lack of sufficient time for students to wrestle with the demanding aspects of the tasks; and classroom management problems that prevent sustained engagement in high-level cognitive activities.

The authors of this study, along with a third individual, served as the primary coders in the initial study. A representative subset (25% of the 144 tasks) were double coded. Consensus was reached by the two coders on all disagreements. Intercoder reliability ranged from 53% to 100%, with an average of 79%.

Sampling for the Present Study

In the initial study, 58 of the 144 tasks were identified as being set up to encourage doing mathematics. These 58 tasks constitute the database for the present investigation. During the implementation phases of these 58 tasks, students were observed to engage actively in doing mathematics in 22 of the tasks. In the remaining 36 tasks, students' engagement with the task during implementation was not observed to exemplify doing mathematics. In 8 tasks, students' thinking focused on procedures without connections to underlying meaning; in 11 tasks, students engaged in unsystematic exploration; and in 10 tasks, students' thinking was perceived to have no mathematical focus. In the remaining 7 tasks, students' forms of thinking during the implementation phase represented a variety of categories of cognitive engagement, no one of which was well enough represented to justify its inclusion in the present reporting.

Analysis Procedures for the Present Study

In order to examine the factors that were associated with maintenance or decline of the doing-mathematics tasks, we first aggregated and summarized the relevant subset of the factors data from the initial investigation according to the maintenance and decline categories identified above. Within each of these maintenance and decline categories, the number of tasks for which each factor was judged to be an influence was calculated. From this information, frequency graphs were constructed in order to be able to identify factor profiles (i.e., the sets of factors judged to be predominant influences in the largest percentage of tasks within each pattern).

After identifying the factor profiles, we returned to our database of narrative summaries and observation videotapes to select classroom episodes that could be developed into portraits exemplifying (a) the maintenance of high-level task demands and the factors that support them and (b) each of the three identified patterns of decline and the factors that influence them. After selecting the classroom episodes, we developed detailed portraits describing the nature of the mathematical task in each episode, how the task was set up by the teacher, how it was implemented by the students, and how the identified factors influenced the implementation of the task.

RESULTS

Maintenance of High-Level Cognitive Demands

Figure 2 shows the percentage of tasks in which each factor was judged to be an influence in assisting students to engage in doing mathematics for the 22 tasks that remained at that level during the implementation phase. The numerals at the top of each bar indicate the number and percentage of tasks for which the particular factor was judged to be an influence in maintaining cognitive demands at the level of doing mathematics (e.g., scaffolding was judged to be an influence in 73% of the tasks). Typically, three to five factors per task were believed by the coders to be influences in assisting students to remain engaged in doing mathematics in particular tasks.

As shown in Figure 2, five factors appeared to be prime influences associated with maintaining student engagement at the level of doing mathematics: task builds on students' prior knowledge (82%), scaffolding (73%), appropriate amount of time (77%), modeling of high-level performance (73%), and sustained press for explanation and meaning (77%). These findings suggest that when tasks successfully maintain students' engagement in doing mathematics, these factors would frequently be expected to be in place supporting that high-level engagement in the tasks.

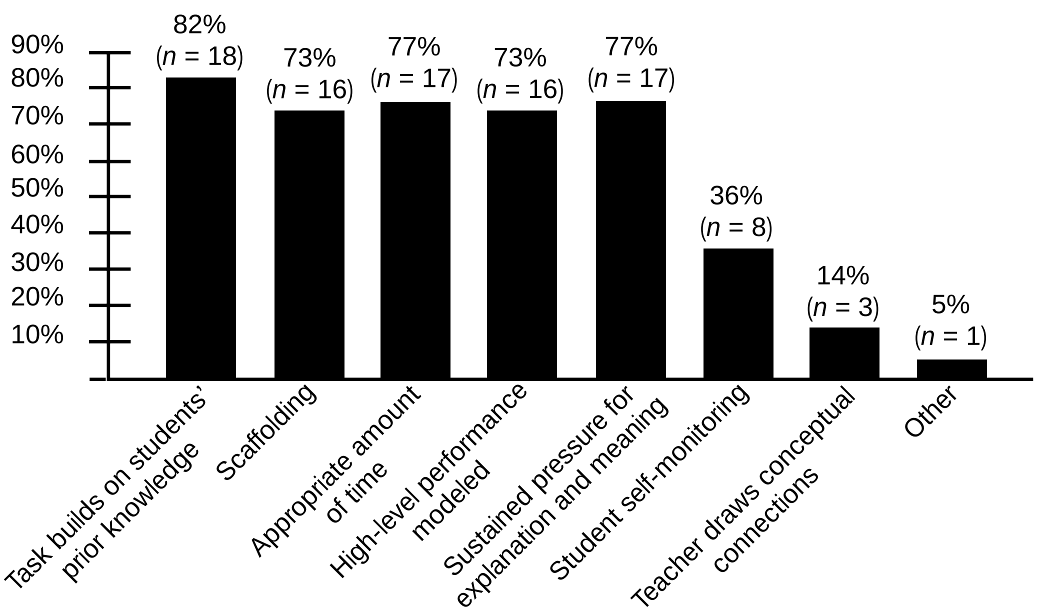


Figure 2. Percentage of tasks on which each factor was judged to be an influence in assisting students to engage at high levels (total number of tasks = 22; percentages total more than 100 because typically more than one factor was selected for each task)

These findings are in agreement with the more general literature on academic tasks outlined earlier. Research has suggested that tasks that are likely to maintain high-level cognitive demands are tasks that build on students' prior knowledge (Bennett & Desforges, 1988) and are allotted an appropriate amount of time for the students to engage at a high level, that is, neither too little nor too much time (Doyle, 1986). Teaching behaviors that were found to support high-level student engagement in this study, including scaffolding, modeling high-level performance, and consistently pressing students to provide meaningful explanations, have also been identified by other researchers as important influences in tasks that encourage students to engage at high levels (Anderson, 1989; Doyle, 1988). These findings demonstrate that even though students were actively engaged during the tasks (as opposed to being passive recipients), teachers still had an important role to play in proactively supporting students' high-level engagement. Two other factors that have been identified in the literature as influential in high-level engagement, students' self-monitoring and frequent conceptual connections drawn by the teacher, were judged in this study to have less influence; that is, they were factors in only 36% and 14% of the tasks, respectively.

Factor Profiles for Specific Patterns of Decline

The factors that were associated with each of the three types of decline are illustrated in Figure 3. In this section, we begin by describing the characteristic factor profiles for each of the three types of decline. We then look across the three profiles of factors to identify their similarities and differences.

Decline into using procedures without connection to concepts, meaning, and understanding. The factors most frequently judged to influence those tasks in which students' thinking processes declined into the use of procedures without connection to meaning or understanding were the removal of challenging aspects of the tasks, shifts in focus from understanding to the correctness or completeness of the answer, and inappropriate amounts of time allotted to the tasks. Of these, the factor most often cited was that the challenging aspects of the task were removed during the implementation phase, thus necessitating lower and less sustained levels of thinking, effort, and reasoning by students. Because high-level tasks can be perceived by students (and teachers) as ambiguous, risky, or both, there is often a "pull" toward reducing their complexity so as to manage the accompanying anxiety (Doyle, 1988). Reduction in complexity can occur in several ways, including through students' successfully pressuring the teacher to provide explicit procedures for completing the task or the teacher's "taking over" difficult pieces of the task and performing them for the students. When this is done, however, the cognitive demands of the task are weakened and students' cognitive processing, in turn, becomes channeled into more predictable and (often) mechanical forms of thinking.

Another frequently cited factor was a classroom-based shift in focus away from meaning and understanding toward the completeness or accuracy of answers. The desired outcome of the task becomes defined by the solution rather than by the thinking processes entailed in reaching the solution. Previous mathematical experiences of both teachers and students often lead to such a narrow preoccupation with solutions, at the expense of understanding. This orientation can easily overwhelm tasks that were initially set up to encourage doing mathematics, especially if a focus on process leads to a slowed instructional pace and lack of complete participation by all. Finally, tasks that decline into procedural forms of student thinking often do so because either too much or too little time is devoted to them. In this situation, students have too little time to grapple with the important mathematical ideas contained in the task. A quick pace gives the impression of covering much ground in an efficient manner but often robs students of the time needed to truly engage with the content and to explore and think in ways characteristic of doing mathematics.

Decline into unsystematic exploration.[5] The factors most frequently judged to influence the decline of tasks into unsatisfying forms of mathematical exploration

[5]The reader should recall that unsystematic exploration refers to students' thinking processes characterized by unsystematic exploration and lack of sustained progress in developing meaning or understanding.

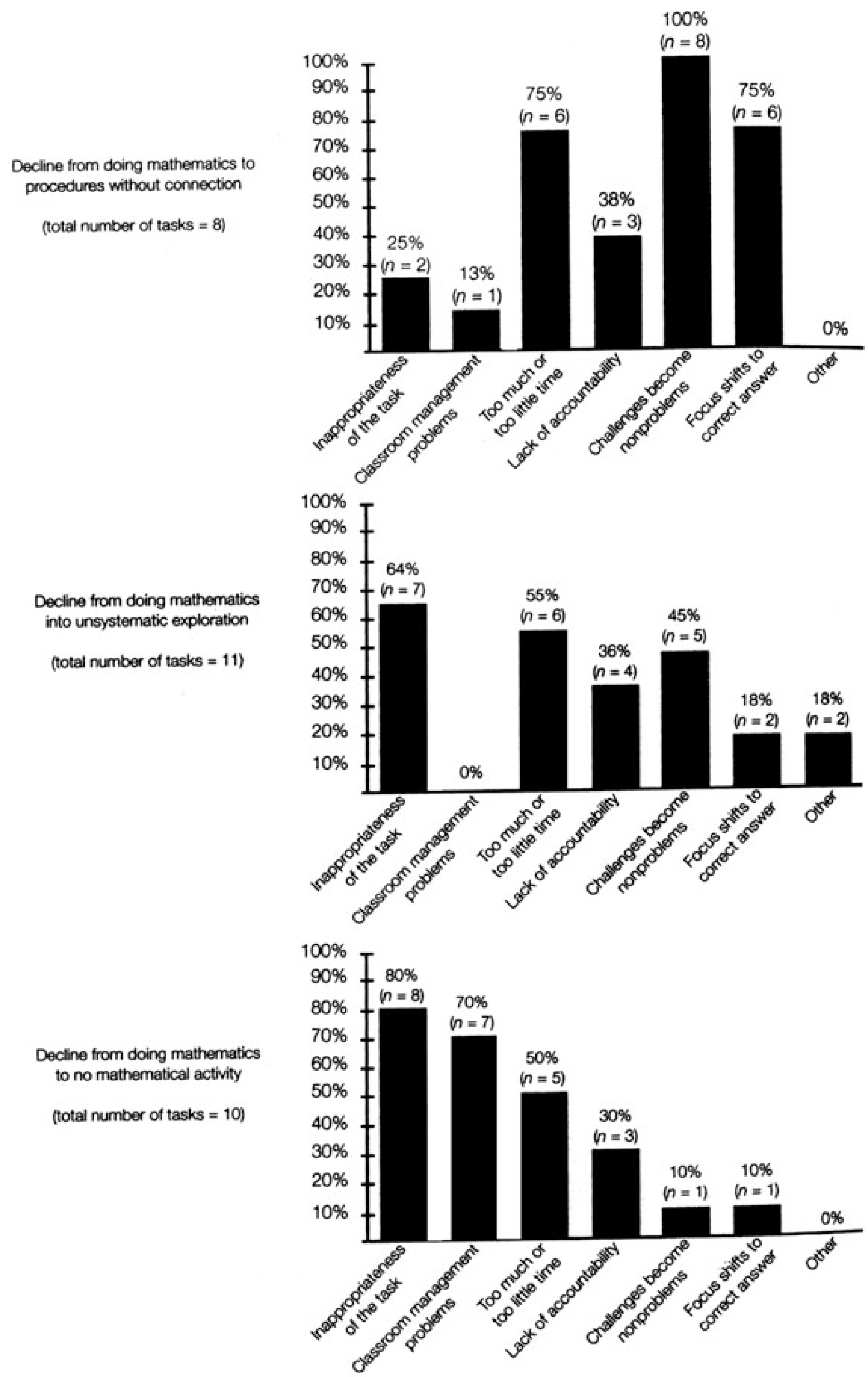


Figure 3. Percentage of tasks in which each factor was judged to be an influence in decline. Percentages total more than 100 because typically more than one factor was selected for each task.

were inappropriateness of the task for the particular group of students, inappropriate amounts of time allotted for those tasks, and the removal of challenging aspects of the tasks. The factor of inappropriateness of the task spans a variety of reasons, including low levels of motivation, lack of prior knowledge, and lack of suitably specific task expectations. All these relate to the appropriateness of the task for a given group of students. As such, they suggest that an important factor in the success of high-level tasks is the consideration of the relationship between students and task; teachers must know their students well in order to make intelligent choices regarding the motivational appeal, difficulty level, and degree of task explicitness needed to move students into the right cognitive and affective space so that high-level thinking can occur and progress can be made on the task.

The second most frequently cited factor was inappropriate amounts of time. In contrast to declines into proceduralized activity, for this type of decline (into unsystematic exploration) the problem was too much time in the majority of tasks for which this factor was judged to be influential. When students are observed not to be making discernible headway toward constructing and understanding key ideas, additional time by itself (i.e., without the introduction of additional support factors) appears to exacerbate the situation. Finally, tasks were observed to decline into unsystematic exploration because the challenging aspects of the task were removed. In these cases, however, the removal of the challenge was less often due to the imposition of a procedure and more often due to the subtle alteration of the task in such a way that the main point of the activity was lost or overshadowed.

Another factor that contributed to the decline of tasks in this category, although less so, was a lack of accountability for high-level products or processes. For example, students were not expected to justify their methods, their unclear or incorrect explanations were accepted, and they were given the impression that their work on these tasks would not “count.” In such instances, students circumvented the “real” tasks and tended to focus only on the work for which they received a grade.

Decline into no mathematical activity. The factors most frequently judged to influence the decline of tasks into activity with no mathematical substance were inappropriateness of the task, classroom management problems, and inappropriate amounts of time. Interestingly, classroom management problems appeared to play a large role when tasks declined into a complete lack of mathematical engagement on the part of the students. This suggests that teachers were struggling with keeping students under control in addition to keeping them focused on the mathematics (although the two may be subtly interrelated). Once again, inappropriate amounts of time were cited, and, in this instance, the problem was too much time in the majority of tasks in which this factor was cited as influential.

Across the three factor profiles, the decline into procedural thinking appears to be associated with the most clearly discernible, “crispest” pattern of factors. The predominance of the three main factors (compared to the relatively weak presence of the other factors) suggests a clear picture of activity in classrooms in which these types of decline occur. Such sharp distinctions among the predominance of the various factors are not as readily identifiable in the declines into unsystematic exploration. In fact, this profile is the least crisp of all, with several factors contributing moderately to the declines. This suggests that there is a less readily apparent set of influences operating in these cases. Given that declines into unsystematic exploration were not anticipated in our initial coding system, the lack of a crisp profile may reflect inadequacies in our factor categories with respect to their ability to capture the kinds of classroom scenarios that lead to these types of decline. The factor profile for declines into no mathematical activity is more sharply differentiated, but not as crisp as the profile for declines into proceduralized forms of students' thinking. The most notable feature of this profile was the strong presence of the factor of classroom management problems.

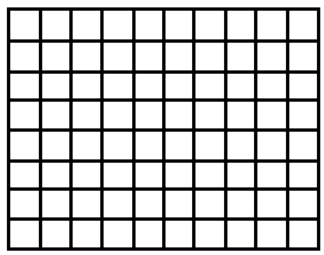
The one major factor that occurred across the three types of decline was inappropriate amounts of time. Thus, it appears as though planning for appropriate amounts of time and being flexible with timing decisions as the task implementation phase unfolds are extremely important in order to avoid declines of all types. Major factors appearing in two out of three profiles were the removal of challenging aspects of the tasks (declines into proceduralized thinking and unsystematic exploration) and inappropriateness of the task for a particular set of students (declines into unsystematic exploration and into no mathematical activity). As mentioned earlier, the factor "removal of challenging aspects of tasks" had slightly different forms in these two types of declines. The factor of inappropriateness of the task was very broad, covering a variety of reasons for low mathematical engagement, all of which related to the appropriateness of the task for a particular group of students.

Qualitative Portraits

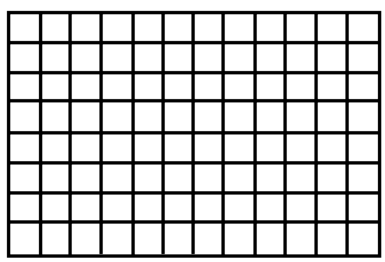
Our final objective is to provide illustrative, qualitative portraits of instruction that represent the factor profiles we found to be associated with the instructional patterns already described. In constructing the portraits, we did not limit ourselves to discussing the specific mathematical tasks from a task-analytic perspective. Instead, we were cognizant of the importance of considering the overall learning environment, including the actions and interactions of the teacher and the students present in the classroom.

Maintaining cognitive demands at the level of doing mathematics. The overall goal of this sequence of lessons was to explore relationships among fractions, decimals, and percentages. Prior to this lesson, students had experienced modeling fractions, decimals, and percentages using multiple representations, and this particular task represented an extension of that work. In the set-up of this task, each student had a set of rectangular grids of various sizes (see Figure 4) and was expected to shade a specified portion of the rectangular area. The portions to be shaded were specified in a variety of ways including a percentage of the total, a fraction of the total, a decimal fraction of the total, or a specific number of squares. Students were then expected to provide for each shaded region a fraction, a decimal, or a percentage that represented the amount of the total area shaded. Also, in a whole-class setting, students were expected to be able to explain one or more solution strategies for each problem. As set up, the task provided an opportunity to facilitate students' construction of connections among the three modes of representing fractional quantities (fractions, decimals, and percentages) in the context of exploring, listening to, interpreting, justifying, and explaining a variety of solution strategies for the problems. Overall, the setup of the task was clearly oriented toward doing mathematics. During implementation of the task, the high cognitive demands were maintained and a variety of factors came into play to support students' high-level engagement with the task.

1. a. Shade .725 of the area of this rectangle.
   b. What fractional part of the area is shaded?
   c. What percentage of the area is shaded?



2. a. Shade $^3/_8$ of the area of this rectangle.
   b. What percentage of the area is shaded?
   c. What decimal part of the area is shaded?



3. a. Shade six of the small squares in this rectangle.
   b. What fractional part of the area is shaded?
   c. What decimal part of the area is shaded?
   d. What percentage of the area is shaded?

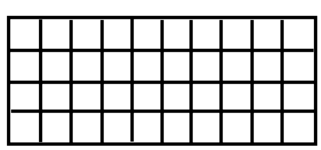


Figure 4. Problems composing the mathematical task of exploring relations among fractions, decimals, and percentages (Bennett & Foreman, 1990)

The foremost influential factor was that the task was designed to build upon students' prior knowledge and experiences with decimals, percentages, and fractions represented in multiple ways. Students had experienced a previous lesson on relating percentages to length and area using a variety of representations, including rectangular areas of various sizes. Thus students were not stymied by the presentation of areas other than the usual 10×10 grid. Mr. Hernandez, the teacher, often directed students' attention to their prior experiences. For example, as a strategy for solving Problem 2a, one student presented a solution in which he regrouped all the squares into eight "piles" and shaded three of the piles. To help students decide what percentage and decimal parts were shaded (Problems 2b and 2c), Mr. Hernandez encouraged them to reason about some basic decimal conversions they already knew. This encouragement led students to use the facts that $^1/_4 = {}^2/_8 = 25\%$ and therefore that $^1/_8 = 12.5\%$. So, $^3/_8$ would have to equal 37.5%, which would be written as .375. In the discussion of Problem 3, Mr. Hernandez encouraged students to use their prior knowledge to convert $^6/_{40}$ to a percentage without using a calculator. One student changed the fraction to $^{12}/_{80}$ and reasoned that in order to get to 100 from 80, he had to add 20, which was $^1/_4$ of 80, so he also added $^1/_4$ of 12 to the numerator to obtain a fraction of $^{15}/_{100}$ or 15%. Another student changed the fraction saying, "If you have 6 in 40, divide it in half, and you'd have 3 in each part; you'd have 3 in 20." He then multiplied the numerator and denominator by 5 to obtain the fraction $^{15}/_{100}$, or 15%.

Another key factor in the students' successful implementation of the task was the scaffolding behavior exhibited by the teacher. Mr. Hernandez was able to assist students as they reasoned through the problems without reducing the complexity of the task at hand. For example, Mr. Hernandez called on Michelle to do Problem 1 at the overhead. In demonstrating Problem 1a, Michelle shaded in 72.5 of the 80 squares. Mr. Hernandez did not immediately correct Michelle's error; instead, he asked her to explain her thinking, but she said she was unsure. When Mr. Hernandez asked her to reread the problem, she realized that she might have made an error; as discussion of Problem 1c ensued, Mr. Hernandez asked how the class could use the information that there were a total 80 squares. He also asked the students to think about what would happen if they tried to distribute 100% across the 80 squares. Michelle thought about it and replied that they could find out what percentage each square represented and that each square would have to be more than 1%. Another student, Cecily, thought each square would be worth 1.25%. Michelle explained further that there would be 20 left over after allotting 1 to each square and that 20 divided among the 80 would give $^1/_4$ more for each square. Michelle was then able to show how many squares should be shaded for 72.5 %. Thus, Mr. Hernandez was able to direct Michelle's (and the class's) attention to appropriate aspects of the task that would enable Michelle to succeed on her own.

At least three other important factors that supported students' engagement in doing mathematics were evident. First, Mr. Hernandez allowed an appropriate amount of time for discussion of the problems, thus affording students opportunities to consider and discuss multiple solution strategies for the problems given. For example, following Michelle's explanation (described above) for Problem 1, another student multiplied .725 by 80 to get 58 and explained that he obtained a fraction of $^{58}/_{80}$ and reduced it to $^{29}/_{40}$. Another student said that she divided the 80 squares into 10 equal columns of 8 squares each and then shaded in 7 columns and 2 more squares (because 2 is $^{1}/_{4}$ of $^{1}/_{10}$ [of 80], or $^{1}/_{4}$ of 8 [in this case], which equals .025) for a total of 58 squares. Another student explained how to use a calculator to find the solutions.

Another factor evident in the lesson was modeling of high-level performance. This factor is well-illustrated by the examples described above in Problem 1. The discussion of multiple solution strategies at the overhead projector provided an opportunity for Mr. Hernandez, as well as several students, to model high-level performance and to make their thought processes explicit. Finally, throughout all the discussion of the problems, Mr. Hernandez pressed students to explain their solution processes. He consistently required students to attach meaning to the symbols and representations within the context of the problems they were solving. When students arrived at a numerical answer, Mr. Hernandez would ask such questions as, "Can you explain what that number refers to?"

This scenario illustrates the variety of classroom, task, teacher, and student factors that supported students' thinking at the level of doing mathematics during the task. As the task unfolded, Mr. Hernandez was able to orchestrate successfully the complex array of factors needed to maintain the high-level cognitive demands of the task.

Decline to procedural thinking without connection to meaning. This mathematical task was embedded in a series of lessons that focused on problem solving. The students had been introduced to Pólya's four-step problem-solving process and had worked on at least one other nonroutine problem before encountering the following:

> For Mother's Day, Davie, my little brother, Kathy, my younger sister, and I all contributed money to buy a present for Mom. Davie had saved 80 pennies, 2 nickels, and 1 dime. Kathy gave me 3 half-dollars that she had saved, and I contributed the rest. Actually, with what Davie and Kathy gave me, the 17 coins in my bank were just enough to make up the total cost of $8.12. What coins were in my bank? (Meyer & Sallee, 1983, p. 335)

Students began working on this task in small groups near the end of one lesson. There appeared to be a clear expectation that the students would work collaboratively to solve the problem. While the teacher, Ms. Capra, set up the task, she provided each group with one copy of the problem and one copy of a form listing Pólya's four problem-solving steps. Students were instructed to refer to Pólya's steps as they solved the problem and to document their work accordingly. The task set-up encouraged cognitive processes that are consistent with doing mathematics. Solving the task demanded complex and sustained mathematical thinking and reasoning, because it could not be solved with well-rehearsed, easily accessible formulas or procedures. Awareness of the Pólya problem-solving steps also may have encouraged self-monitoring and regulation of students' own thought processes, a hallmark of high-level thinking.

During task implementation, students' cognitive processes declined into procedural thinking that made little if any connection to understanding or meaning. Students' failure to engage in high-level cognitive processes was influenced by a variety of factors, the foremost being the removal of challenging aspects of the problem. Specifically, the teacher's orchestration of students' engagement with the task in a manner that strictly followed Pólya's four-step process channeled their thinking into predetermined pathways at the expense of their grappling with the mathematical ideas and processes embedded in the task. Throughout task implementation, Ms. Capra regulated students' thinking processes by partitioning the task into steps that matched each step of the Pólya process. First, students were instructed to work on the first Pólya step (understanding the problem) in their small groups and then to discuss it in a whole-class discussion. Then they were told to do the same with the second Pólya step (devising a plan). It was only after completing these initial two steps that students began to work on solving the problem within their small groups. At any given point in time, students' work was constrained by the products associated with a particular step in the Pólya process.

Within each of these steps, there was evidence that students were dealing with Pólya's ideas at a superficial level. For example, while Ms. Capra and her students discussed Step 1, understanding the problem, they focused on listing the main facts of the problem (e.g., Davie contributed 80 pennies, 2 nickels, and 1 dime). They did not advance to a discussion of the main mathematical features of the problem, the structure of the relationships among those features, or the goal of the problem. Similarly, during discussion of Step 2, a label of a strategy was stated (guess and check), but no discussion took place regarding what the strategy entailed, why it was an appropriate approach for the present situation, or how to use the strategy.

The decline into mechanical forms of thinking was also influenced by a focus on correct answers. In this case, the answer that was judged as correct or incorrect was not the *mathematical* solution to the problem, but rather the students' responses to Pólya's four-step process. The "correctness" of students' responses to each of these steps became the focus, rather than the validity of their mathematical approach to the problem.

Finally, the decline of students' engagement with high-level cognitive processes was also influenced by lack of time. Task implementation moved very quickly, progressing from one step to the next with very little time spent on any one aspect of the problem-solving process. At each step, the students' small-group discussions were halted prematurely as specific students were asked to display their responses to the entire class. Students had only 10 minutes to work on solving the Mother's Day problem (Step 3) in their small groups. When Ms. Capra said it was time to stop, many groups appeared to need and to desire more time.

Overall, students' work on this potentially rich task did not progress into meaningful engagement with the mathematical content and processes embedded in the problem. By fragmenting and channeling students' thinking, the use of Pólya's steps (along with the rushed nature of the class) in this case, led to a mechanical, as opposed to substantive and creative, engagement with the problem.

Decline to unsystematic exploration. The overall goal of this sequence of lessons was to have students explore metric area measurement, as well as relationships between linear and area measurement. Students had previously experienced a series of lessons involving linear and area measurement with nonstandard units and were just beginning to explore metric area measure. One focus of the previous lessons had been on the relationships among different units of measure in the context of linear versus area measurement.

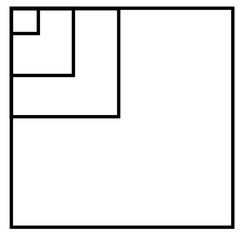
In the task setup, each group of three or four students received a teacher-made activity sheet[6] explaining that their first task was to build a square meter. They were also to construct the square meter as efficiently as possible using any of the following materials (but no scissors): paper, tape, rulers, and base-ten pieces to measure. After constructing the square meter, students were to model a square decimeter, a square centimeter, and a square millimeter in one corner of their square-meter model (placing smaller squares inside the larger squares). By constructing the square meter themselves, students could gain a good sense of the size of the square meter and how its size related to other metric area units. In addition, students could use their models to explore other rectangles with the same area (by cutting up and rearranging their square-meter models). At set-up, the task encouraged students' engagement in doing mathematics. In executing the task, students were required to rely on knowledge gained from their prior measurement experiences and to decide what units of measure and measuring tools to use, given the limitations set by their teacher, Ms. Hoffman. Also, students had to make conjectures about how to build the square meter efficiently and accurately and how to show the relationships among the different-sized squares.

As implementation of the task unfolded, it became clear that even though the students were on-task, the cognitive demands of the task were not maintained. As evidenced by their construction methods, many groups appeared to be viewing their squares solely in terms of linear dimensions, with no focus on area units. Many groups built their models by first constructing four strings of paper each of length 1 meter, then forming a square, and finally filling in the empty space in the middle (essentially building the entire perimeter first). Also, the students' lack of engagement in the intended high-level processes was evident in the way many groups interpreted the instructions for modeling the smaller metric squares. They seemed to have missed the point, drawing the smaller squares as isolated models, one inside the other. Ms. Hoffman intended for students to draw the smaller squares so that each square shared both area and partial linear dimensions with the squares larger and smaller than itself (see Figure 5 for an illustration), making visually apparent the relationships among the different metric linear and area measures.

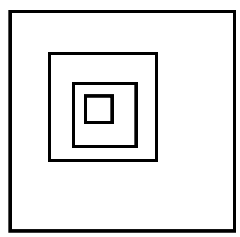
One reason students failed to engage with the intended processes was the inappropriateness of the task with respect to the clarity and specificity of task expectations, as evidenced by the large amount of time most groups spent completing their constructions and by the fact that students seemed to need more guidance about what was expected in terms of how to situate the smaller squares within their square meter models. Ms. Hoffman could have instructed students to place the smaller square areas within the larger area so that the relationships among the linear and area measures would have been evident. This instruction would have provided a little more direction, while maintaining the cognitive demands of the task at that point. An additional

[6]The teacher adapted and expanded suggestions in Teacher Activity 109B found in Bennett and Foreman (1990).

reason, related to the inappropriateness of the task for this group of students, appeared to be their lack of prior knowledge needed to attack the problem efficiently. For example, many students lacked knowledge about relationships among different-sized linear dimensions and especially about how linear dimensions could be related to area.



An arrangement that illustrates how
the lengths and areas are related



The type of arrangement constructed
by many of the students

Figure 5. "Build a square meter" task (*Note*. Figures are not drawn to scale)

Another factor that influenced the decline in cognitive demands was that students may have been given too much time to build their square meters. Students were given the entire period to complete the activity. If there had been a tighter time limit, some groups may have been motivated to work more efficiently. Also, since many of the groups appeared to be floundering with this part of the task, it may have been helpful for the teacher to intervene with these groups early in order to help them adopt a more efficient plan of action.

A final factor that influenced the decline of this task was that some of the mathematically challenging aspects of the task were removed or became overshadowed in the lengthy process of building the square meter. The groups that reached the part of the task in which they were supposed to draw in the different-sized metric areas did not seem to be frustrated by it. The ways in which many groups did this (shown in Figure 5), however, indicated that they did not realize the significance of the placement of these squares. Thus, this part of the task became a nonproblem for the students, simply a matter of drawing some different-sized squares inside one another.

Despite the fact that the majority of students were actively engaged in the lesson overall, they failed to focus on the important mathematical ideas contained within this complex and interesting task. Students earnestly tried to engage in doing mathematics, and the teacher earnestly tried to support the maintenance of the task at a high level; however, for the majority of students, these efforts were unsuccessful. The difficulties did not lie in classroom management problems, and the teacher never shifted the focus to one correct way of doing the task. Instead, the cognitive demands declined primarily because of several factors related to the appropriateness of the task for the students and the level and kind of guidance they needed to engage at the level of doing mathematics.

Decline to no mathematical activity. The overall goal of this sequence of the lessons was to encourage students to use pattern finding to discover properties of two-dimensional geometric figures rather than to memorize their properties. This particular task focused on angles and how they can be used to define different types of triangles.

In the task setup, Mr. Kingsley, the teacher, gave each pair of students a tangram puzzle consisting of five triangles, one square, and one nonrectangular parallelogram arranged in such a way that the shapes covered the entire area of a square. The students were also given an activity sheet asking them to systematically explore the similarities and differences in the angles of the pieces. Students were expected to (a) identify the two pieces that were the same as (congruent with) other pieces and eliminate them, (b) record the type of each angle in the remaining five pieces, and (c) examine and record similarities and differences in the angles across those five pieces. This systematic exploration was ultimately to result in students' recognition that the triangular tangram pieces were right isosceles triangles and that the triangles' acute angles were congruent with the acute angles in the parallelogram piece. As set up, the task encouraged students to engage in cognitive processes that are consistent with doing mathematics, such as discovering important mathematical ideas through hands-on exploration of patterns, establishing and implementing a systematic method of recording the results of one's explorations, and making observations of similarities and differences across the angles of different geometric figures to deepen understandings of connections among various geometric shapes.

Students did not make much, if any, progress on this task. They were most involved with the task when Mr. Kingsley was at their table assisting them and asking them questions. During the rest of the time, they were only half-interested, sat idle, played with the tangram pieces in nonmathematical ways, or talked to other students in the class about nonmathematical topics.

The students failed to engage in the intended high-level cognitive processes for a variety of reasons. The foremost reason was the inappropriateness of the task with respect to the clarity and specificity of the task expectations. Task expectations were not specific enough to guide students toward discovering the relevant mathematical properties. The lack of specificity was especially problematic because the students lacked the relevant prior knowledge needed to make effective comparisons and differentiations. For example, students' inability to distinguish acute, obtuse, and right angles hindered their efforts to record systematically and generalize their findings. As a result, most students played around with a few comparisons but failed to make progress systematically in discovering the ways in which all the triangles were similar.

Another factor that led to a decline in the cognitive demands on the students was classroom management problems. At the beginning of the implementation phase, some of the students needed pencils, others needed recording sheets to record the information they discovered, and still others were working or playing with the tangram pieces. Throughout the task, a significant subset of students wandered freely around the room, visiting with their friends. Many students appeared to be engaging in off-task behaviors because they were unsure how to proceed with the task.

The final factor that contributed to the decline in the level of students' engagement with this task during the implementation phase was the amount of time that students were allowed to flounder. Despite the fact that little progress was being made, students were allowed to continue work on this task for 38 minutes. The amount of off-task behavior increased steadily during this time as students appeared to reach the conclusion that they could not work effectively on the task.

Overall, the decline into lower levels of student engagement with this potentially rich task was representative of the ways in which other high-level tasks in our database declined into no mathematical activity. In this particular case, classroom management and timing problems appeared to be closely intertwined with the problem of mismatch between the cognitive demands of the task and students' prior knowledge. Although both the lack of attention to appropriate tasks and the recording systems that contributed to the management problems appeared from the outset of the task, it was difficult to discern which of these problems came first.

SUMMARY AND CONCLUSIONS

At the outset of the study, we sought to address three areas: (a) to provide a profile of factors associated with tasks that were set up to engage students in cognitive processes at the level of doing mathematics and that did engage students in high-level thinking and reasoning, (b) to describe factors influencing the three characteristic patterns of decline in students' engagement with high-level cognitive processes, and (c) to provide detailed qualitative portraits from our database to illustrate all four patterns and the profiles of factors associated with them.

Our findings suggest that there was a discernible set of factors influential in assisting students to engage at high levels. These included factors related to the appropriateness of the task for the students and to supportive actions by teachers, such as scaffolding and consistently pressing students to provide meaningful explanations or make meaningful connections. These findings have implications for the role of the teacher in reform classrooms, in which students are expected to be actively engaged in doing mathematics. Not only must the teacher select and appropriately set up worthwhile mathematical tasks, but the teacher must also proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task.

Students' engagement with tasks that declined to lower levels of cognitive activity happened in different ways and for different reasons. For each of the three patterns of decline, we were able to identify a set of predominant classroom-based factors that contributed to the decline in the cognitive demands of the tasks;

however, there was variation in how sharply distinguishable the factor profiles were across decline patterns.

The least readily identifiable factor profile was associated with decline to unsystematic exploration, a pattern that was not anticipated in our initial coding system. In this pattern, the students earnestly attempted to remain faithful to the setup of the task at the level of doing mathematics and teachers made attempts to support high-level engagement, but students were ultimately unsuccessful with respect to performing at a high level and engaging with the important mathematical ideas in the task. As we pointed out earlier, the lack of a crisp factor profile associated with this pattern may be due to inadequacies in our factor categories. Unlike the other two patterns that reflected a decline from doing mathematics to some other specific level of students' engagement with cognitive processes, this pattern declines *from* doing mathematics, but does not reflect a decline to a readily identifiable level of students' cognitive activity. We speculate that as teachers and students become more confident and more willing to take risks with the kinds of tasks that aim to engage students in doing mathematics, this type of decline pattern might become more prevalent than other types of decline in students' engagement. This conjecture is an empirical question that may warrant further investigation.

Across all three patterns there was one factor, the appropriateness of the amount of time (either too little or too much) allotted for the task, that appeared as a predominant influence; however, this factor appeared to function differently in each of the three decline patterns. In agreement with research on students' engagement with academic tasks, these findings suggest that planning for appropriate amounts of time and flexibility with timing decisions may play an important role in avoiding decreases in the level of cognitive activity engaged in by students as tasks unfold in the classroom (Doyle, 1986). Two other factors, removal of challenging aspects of the task and inappropriateness of the task for a variety of reasons (e.g., lack of interest, motivation, knowledge, or unclear task expectations), were each judged to be predominant influences in two of the three decline patterns. The prevalence of these factors as influences in students' declining cognitive activity has also been found by other researchers (Bennett & Desforges, 1988; Doyle, 1983, 1986).

The results enabled us to examine also factors that were not judged to be influential in the three characteristic patterns of decline in students' engagement. We found that classroom management problems were a predominant influence in only one of the three characteristic patterns: decline to no mathematical activity. In the other two patterns combined, classroom management was an influential factor in the decline of only one task, a finding that seems to be at odds with the more general literature on academic tasks; this literature suggests that classroom management problems often affect the implementation of high-level tasks (Doyle, 1988). Future research could investigate more deeply the variations in how classroom management did or did not influence the different patterns of decline in the tasks available in our database.

The use of real classroom-based scenarios to illustrate empirically generated patterns of students' engagement can be seen as similar to the use of cases to illuminate general principles. More generally, within the field of research on teaching, Shulman (1986) has discussed the need for the development of specific cases that are connected to more general principles of instruction. According to Shulman, such cases are useful because they connect to the complex world of everyday practice and also to a larger set of ideas about instruction. These larger ideas have the disadvantage of being abstract, but at the same time, they have the advantage of being more generalizable. When cases are selected or developed to illustrate principles or ideas, the resultant product has the advantage of being not only an interesting account of practice, but also a case of a particular principle. By being related to larger sets of ideas, the cases become more meaningful, more "connectable" to other important ideas, and more powerful as a guide for future research or practice.

In this study, the patterns of student engagement and profiles of influential factors, although not principles, did provide a coherent framework within which the qualitative portraits could be interpreted. Students' engagement patterns and the factor profiles had certain qualities of abstraction and generalizability because they had been suggested by patterns of findings from an earlier empirical investigation. When portraits are placed into this larger conceptual space, their meanings become more readily apparent. Individual actions of teachers and students can be interpreted within a frame of reference that includes more general notions of pedagogy and the development of mathematical understandings by students.

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