7.6 MATHEMATICS

The task of instruction in mathematics is to offer opportunities for the development of mathematical thinking, and for the learning of mathematical concepts and the most widely used problem-solving methods. The instruction is to develop the pupil's creative and precise thinking, and guide the pupil in finding and formulating problems, and in seeking solutions to them. The importance of mathematics has to be perceived broadly: it influences the pupil's intellectual growth and advances purposeful activity and social interaction on his or her part.

Mathematics instruction must progress systematically and create a lasting foundation for the assimilation of mathematical concepts and structures. The discipline's concrete nature serves as an important aid in bringing together the pupil's experiences and systems of thought with the abstract system of mathematics. Problems that come up in day-to-day situations, and that can be resolved with the aid of mathematical thinking or operations, are to be utilized effectively. Information and communication technology are to be used to support the pupil's learning process.

GRADES 1–2

The core tasks of mathematics instruction in the first and second grades are the development of mathematical thinking; practice concentrating, listening and communicating; and acquisition of experience as a basis for the formulation of mathematical concepts and structures.

OBJECTIVES

The pupils will

- learn to concentrate, listen, communicate, and develop their thinking; they will derive satisfaction and pleasure from understanding and solving problems
- gain diverse experience with different ways of presenting mathematical concepts; in the concept formation process, the central aspects will be spoken and written language, tools, and symbols
- understand that concepts form structures
- understand the concept of the natural number and learn the basic computational skills appropriate to it
- learn to justify their solutions and conclusions by means of pictures and concrete models and tools, in writing or orally; and to find similarities, differences, regularities, and cause-and-effect relationships between phenomena
- become practised in making observations about mathematical problems that come up and are challenging and important from their personal standpoints.

CORE CONTENTS

Numbers and calculations

- number, numeral, and number symbol
- properties of numbers: comparison, classification, ordering, using concrete means to break down and assemble numbers
- principle on which the decimal system is based
- addition and subtraction, and connections between calculations, using natural numbers
- multiplication and multiplication tables
- division, using concrete tools
- use of different ways and means of calculating: blocks and decimal tools, continuum, mental calculation, using pencil and paper
- investigating the number of various alternatives
- introducing the concept of the fraction by concrete means

Algebra

- seeing regularities, ratios, and correlations pictorially
- simple number sequences

Geometry

- observing and describing spatial relationships in the surrounding space
- observing, describing and naming geometric shapes in the environment
- recognizing, explaining, and naming two- and three-dimensional figures
- basic geometric concepts such as the point, line segment, horizontal line, ray, straight line, and angle
- making, drawing, and tracing two-dimensional figures, and recognizing and constructing three-dimensional figures;
- simple reflections and dilations

Measurement

- principle of measurement
- length, mass, surface area, volume, time, and price
- use of measuring devices
- use and comparison of the most important units of measurement
- assessment of measurement results

Data processing and statistics

- looking for, collecting, and storing data
- reading simple tables and diagrams
- presenting assembled data as a bar graph

DESCRIPTION OF GOOD PERFORMANCE AT THE END OF THE SECOND GRADE

Thinking and working skills

The pupils will

- demonstrate an understanding of concepts associated with mathematics by using them to solve problems, and by presenting and explaining them to the teacher and other pupils
- be able to reach justified conclusions and to explain what they have done, and know how to present their solutions by means of pictures and concrete models and tools, orally and in writing
- know how to perform comparisons, such as comparisons of length; to place things in order; to find opposites for things; to classify things according to different attributes; to state the location of an object, for example by using the words above, below, on the right, on the left, behind, and between; to compare the size of sets, using the words more, fewer, as many, a lot, and a few; and to write and use the comparative symbols >, =, and <.

Numbers, calculations, and algebra

The pupils will

- know the importance of numbers in stating amount and order; they will know how to write numbers and present a continuum
- master the breaking down and assembly of numbers, comparison, and the formation of sums and number sequences; they will know about odd and even numbers
- know about and understand the decimal system as a place system, and know how to use it
- understand addition, subtraction, multiplication, and division and know how to apply them to everyday situations
- know how to look for the number of alternative solutions in simple events
- know simple fractions, such as one half, one third and one quarter, and know how to present them by concrete means.

Geometry

The pupils will

- know the basic forms of plane and three-dimensional figures, including the quadrangle, triangle, circle, sphere and cube, and know the basic concepts of geometry – the point, line segment, horizontal line, ray, line, and angle – and their relationship to the simplest plane figures
- know how to use simple reflections and dilations.

Measurement

The pupils will

- know how to measure with simple measuring devices, and know the main quantitative expressions, such as length, mass, volume, and time
- be able to note the necessary information in simple, day-to-day problems, and to use their mathematical knowledge and skills to solve those problems.

GRADES 3–5

The core tasks of mathematics instruction in the third through fifth grades are to develop mathematical thinking, introduce the learning of mathematical models of thinking, strengthen basic calculations and the concept of the number, and provide experiences as a basis for assimilating the concepts and structures of mathematics.

OBJECTIVES

The pupils will

- gain experience in succeeding with mathematics
- learn, through investigation and observation, to formulate mathematical concepts and concept systems
- learn to use mathematical concepts
- learn basic calculation skills and learn to solve mathematical problems
- find similarities, differences, regularities, and cause-and-result relationships between phenomena
- justify their actions and conclusions and present their solutions to others
- learn to present questions and conclusions on the basis of observations
- learn to use rules and follow directions
- learn to do sustained, focused work, and to work in a group.

CORE CONTENTS

Numbers and calculations

- strengthening the concept of the decimal system, introduction to the 60-based system with the help of the times on a clock
- classification and organization of numbers
- multiplication
- division in a ratio, division into parts, divisibility
- algorithms and mental calculation
- concept of the fraction, conversion of fractions
- concept of the decimal fraction
- relationship between the fraction, decimal fraction, and percentage
- addition and subtraction of fractions and decimal fractions, and their multiplication and division by natural numbers
- evaluating, checking, and rounding the results of calculations
- use of parentheses
- concept of the negative whole number
- investigating the number of different alternatives

Algebra

- concept of the algebraic expression
- interpretation and writing of number sequences
- regularities, ratios, and correlations
- seeking solutions to equations and inequalities by deduction

Geometry

- dilations and reductions; similarity and scale
- reflections across a line and around a point, symmetry, congruence, using concrete means
- the circle and its parts
- parallel and perpendicular lines
- angle measurement and classification of angles
- study and classification of different types of polygons
- circumference and area
- study of the geometric properties of two- and three-dimensional figures
- reinforcing comprehension of the principle of measurement
- use, comparison, and conversion of units of measurement
- evaluation of measurement results; revision of measurement

Data processing, statistics and probability

- searching for, gathering, storing, and presenting data
- the coordinate system
- reading simple tables and diagrams
- concept and computation of the arithmetic mean
- classification and organization of data; introduction to the concepts of mode and median
- experiences with classical and statistical probability

DESCRIPTION OF GOOD PERFORMANCE AT THE END OF THE FIFTH GRADE

Thinking and working skills

The pupils will

- demonstrate an understanding of concepts associated with mathematics by using them in problem-solving, and by presenting them in diverse ways - with instruments, pictures, symbols, words, numbers, or diagrams
- try consciously to focus their attention when making observations; they will be able to communicate their observations and thoughts in diverse ways - by acting, speaking, writing, and using symbols
- know how to depict real-world situations and phenomena mathematically by comparing, classifying, organizing, constructing, and modelling
- know how to group or classify on the basis of a given or chosen criterion, to look for a shared attribute, to distinguish between a qualitative and quantitative property, and to describe groups of things and objects, positing true and untrue propositions about them

- know how to present mathematical problems in a new form; they will be able to interpret a simple text, illustration, or event and to make a plan for solving the problem
- know how to follow rules.

Numbers, calculations, and algebra

The pupils will

- understand the decimal system in terms of decimal fractions, too, and know how to use it confidently; they will understand the concepts of the negative number and fraction and be able to present them by different methods
- know how to present calculations in writing and orally, and know the relationships between different calculations; they will know how to estimate in advance the magnitude of the result and, after the problem is solved, to check the stages of the calculation and evaluate the sensibleness of the solution
- know how to formulate and continue number sequences and to present correlations.

Geometry

The pupils will

- know how to form figures, following the instructions given; they will be able to notice the properties of simple geometric figures and will be familiar with the structure formed by the concepts of plane figures
- recognize similarity; they will know how to reflect a figure across a line, and to dilate and reduce figures by a given ratio; they will recognize figures that are symmetrical in relation to a line
- understand the principle of measurement; they will know how to evaluate the size of the object being measured and the sensibleness of the measurement's result, and how to state that result in appropriate units of measurement
- know how to calculate the area and perimeter of parallelograms and triangles.

Data processing, statistics, and probability

The pupils will

- know how to gather data and organize, classify, and present them as statistics; they will know how to read simple tables and diagrams
- know how to clarify the number of different events and alternatives, and to judge which is an impossible or certain event.

GRADES 6–9

The core task of mathematics instruction in the sixth through ninth grades is to deepen understanding of mathematical concepts and furnish adequate basic capabilities encompassing the modelling of everyday mathematical problems, the learning of mathematical models of thinking, and practice with remembering, focusing, and precise expression.

OBJECTIVES

The pupils will

- learn to trust themselves, and to take responsibility for their own learning in mathematics
- come to understand the importance of mathematical concepts and rules, and to see the connections between mathematics and the real world
- learn to perform calculations and solve mathematical problems
- learn to think logically and creatively
- learn to apply various methods to the acquisition and processing of information
- learn to express their thoughts unambiguously and to justify their actions and conclusions
- learn to present questions and conclusions on the basis of observations
- learn to perceive regularities
- learn to work in a sustained, focused manner, and to function in a group.

CORE CONTENTS

Thinking skills and methods

- functions that demand logical thinking, such as classification, comparison, organization, measurement, constructing, modelling, and looking for and presenting rules and correlations
- interpretation and use of concepts needed in drawing comparisons and correlations
- interpretation and production of mathematical texts
- introduction to proof: justified conjectures and experiments, systematic trial-and-error method, demonstrating incorrectness, direct proof
- solving combinatorial problems by different methods
- use of tools and drawings that assist thinking
- history of mathematics

Numbers and calculations

- strengthening basic calculation skills
- natural numbers, whole numbers, rational numbers, real numbers
- opposite numbers, absolute values, reciprocals
- time calculations, time intervals
- prime numbers, division of numbers into prime factors, rules for divisibility of numbers
- reduction of fractions, conversion of fractions to higher terms, and presentation of decimal fractions as common fractions
- multiplication and division with fractions, including decimal fractions
- reduction of expressions
- ratio and proportionality
- strengthening the concept of percentage, percentage calculation
- rounding and estimation; using a calculator
- powers using whole-number exponents
- concept of the root, square-root calculations

Algebra

- the expression and its reduction
- the exponential expression and its reduction
- concept of the polynomial; addition, subtraction and multiplication of polynomials
- concept of the variable; calculating the value of an expression
- equation, inequality, domain, solution set
- solving a first-degree equation
- solving an incomplete quadratic equation
- proportion
- pairs of equations and their resolution algebraically and graphically
- study and formulation of number sequences

Functions

- observing correlation and presenting it by means of variables
- concept of the function
- presenting a set of coordinates in a coordinate system
- interpreting simple functions and drawing their graphs in a coordinate system
- investigating the graph of a function: the function's null point, largest and smallest value, increase, and decrease
- the linear function
- direct and inverse proportionality

Geometry

- relationships between angles
- concepts related to triangles and quadrangles
- regular polygons
- the circle and related concepts
- calculating the perimeter and area of plane figures
- naming and classifying three-dimensional figures
- calculating the volume and surface area of a three-dimensional figure
- similarity and congruence
- geometric construction
- depictions of congruence: reflections, rotation, and transformation
- Pythagorean theorem
- relationships between the triangle and the circle
- trigonometry and solving the right triangle

Probability and statistics

- concept of probability
- frequency and relative frequency
- determining the average, mode, and median

- concept of dispersion
- interpretation of diagrams
- gathering and adapting information, and presenting it in a usable form

FINAL-ASSESSMENT CRITERIA FOR A GRADE OF 8

Thinking skills and methods

The pupils will

- notice parallels and regularities between different events
- know how to use logical elements such as and, or, if so, no, exists, and does not exist in their speech
- know how to judge the truth of simple propositions
- know how to transform a simple problem in text form to a mathematical form of presentation, make a plan to solve the problem, solve it, and check the correctness of the result
- know how to use classification in solving mathematical problems
- know how to present possible alternative solutions systematically, using a table, elm tree diagram, path diagram, or other diagram.

Numbers and calculations

The pupils will know how to

- estimate a possible result and prepare a plan for solving a problem; they will have dependable basic calculation skills
- raise a number to a natural-number power and be able to divide a number into its prime factors
- solve problems in which a square root is needed
- use proportion, percentage computation, and other calculations in solving problems that come up in day-to-day life.

Algebra

The pupils will know how to

- solve a first-degree equation
- reduce simple algebraic expressions
- perform calculations of powers
- formulate a simple equation concerning a problem connected to day-to-day life and solve it either algebraically or by deduction
- use pairs of equations for solving simple problems
- evaluate the logic of a result and examine the different phases of their solution.

Functions

The pupils will

- know how to determine the coordinates of a point in a coordinate system
- know how to prepare a table from number pairs according to the rule given

- know how to look for the null point of a linear function
- know how to continue a number sequence according to the rule given and be able to describe the general rule for a given number sequence verbally
- know the meaning of the constant and slope of a line equation; they will know how to determine the point of intersection of two straight lines by drawing them.

Geometry

The pupils will know how to

- recognize different geometric forms and will know their properties
- apply what they have learned to ways of calculating circumference, area, and volume
- use a compass and ruler to make simple geometric constructions
- find similar, congruent, and symmetrical figures and be able to apply this skill in investigating the properties of triangles and quadrangles
- apply the relationships between two angles in simple situations
- use the Pythagorean theorem and trigonometry to solve the parts of a right triangle
- perform measurements and related calculations and convert the most common units of measurements.

Probability and statistics

The pupils will know how to

- determine the number of possible events and organize a simple empirical investigation of probability; they will understand the meaning of probability and randomness in day-to-day situations
- read various tables and diagrams, and to determine frequencies, average, median, and mode from the given material.