INTRODUCTION

The *Michigan Curriculum Framework* is a resource for helping Michigan's public and private schools design, implement, and assess their core content area curricula. The content standards identified in this document are presented as models for the development of local district curriculum by the Michigan State Board of Education and the Michigan Department of Education. They represent rigorous expectations for student performance, and describe the knowledge and abilities needed to be successful in today's society. When content, instruction, and local and state assessments are aligned, they become powerful forces that contribute to the success of student achievement.

The framework presents a content and a process for developing curriculum that enables schools to realize Michigan's vision for K-12 education:

> **Michigan's K-12 education will ensure that all students will develop their potential in order to lead productive and satisfying lives. All students will engage in challenging and purposeful learning that blends their experiences with content knowledge and real-world applications in preparation for their adult roles, which include becoming:**
>
> - ***literate individuals***
> - ***healthy and fit people***
> - ***responsible family members***
> - ***productive workers***
> - ***involved citizens***
> - ***self-directed, lifelong learners***

The intent of this document is to provide useful resources to districts as they strive to implement a program which ensures that all students reap the benefits of a quality education and achieve the adult roles described in Michigan's vision for K-12 education. The content standards and benchmarks serve as worthy goals for all students as they develop the knowledge and abilities inherent in their adult roles. They represent an essential component in the process of continuous school improvement, which like professional development, should be focused on improving student achievement.

We believe that efforts to set clear, common, state and/or community-based academic standards for students in a given school district or state are necessary to improve student performance. Academic standards clearly define what students should know and be able to do at certain points in their schooling to be considered proficient in specific academic areas. We believe that states and communities can benefit from working together to tap into the nation's best thinking on standards and assessments.

1996 National Education Summit Policy Statement

The framework emphasizes the importance of:

- using continuous school improvement to align all district initiatives for the purpose of increasing student achievement;
- building a curriculum based on rigorous content standards and benchmarks;
- using student achievement data to make decisions about continuous school improvement, curriculum, instruction, and professional development; and,
- incorporating research-supported teaching and learning standards into daily instructional practice.

BACKGROUND

In 1993, the Michigan Department of Education, in collaboration with representatives from five state universities, was awarded federal funding from the U.S. Department of Education to develop curriculum framework components for English language arts, mathematics, science, and geography. In addition, the Michigan Council for the Social Studies offered, and was supported by the State Board of Education, to develop a curriculum framework component for social studies which would include history, economics, and American government, and would be complementary to the geography framework.

"Setting high standards for our children. It's the sine qua non for any other reforms anyone might want to implement. We can't judge the efficacy of ideas because we have no yardsticks by which to measure success or failure."

Louis V. Gerstner
Chairman and CEO, IBM

The *Michigan Curriculum Framework* brings together the work of individual content area projects to present a unified view of curriculum, one which addresses the educational needs of the whole learner. The goal of the curriculum framework is to improve student achievement by aligning classroom instruction with core curriculum content standards and national content standards. It is designed to be used as a process for the decision-making that guides continuous school improvement. It describes curriculum, instruction, and assessment and focuses on improving program quality by aligning all the processes that affect a student's achievement of rigorous content standards.

Framework project co-directors, university representatives, and the Michigan Department of Education content area consultants met regularly with members of their content area professional organizations to design the components of the curriculum framework. Committees of teachers and university personnel worked together to draft the content standards, benchmarks, and performance standards for their specific content areas.

Co-directors met on a monthly basis to coordinate the efforts of the various content area committees in developing K-12 standards and benchmarks for their subject areas. Their purpose was to ensure that the framework represents a consistent view of curriculum across content areas. They wanted to facilitate continuous school improvement by emphasizing commonalities among the content areas with regard to professional development, assessment, and instruction.

The co-directors were guided by a Joint Steering Committee comprised of representatives from the content areas, parents, business leaders, labor leaders, house and senate staff, and educators. Joint Steering Committee members reviewed the framework projects at each phase of their development and made recommendations for improving their quality. Their insight helped the co-directors incorporate the views of all of Michigan's interested parties into the final framework document.

WHAT IS IN THE FRAMEWORK?

The framework includes the resources needed to develop a standards-based curriculum. Standards and benchmarks for English language arts, mathematics, science, and social studies are included in this edition of the framework. Standards and benchmarks for arts education, career and employability skills, health education, life management education, physical education, technology education, and world languages will be added to the next edition of the framework. The process described in the framework will be expanded to incorporate the additional core content areas when they are completed. The chart on page vi provides a list of the materials that eventually will be included in the framework document.

Tier I

Content Standards and Benchmarks

Tier I begins with a complete list of core curriculum content standards and benchmarks for grades K-12 in the areas of English language arts, mathematics, science and social science. The standards describe what all students should know and be able to do in each of the subject areas. The benchmarks indicate what students should know and be able to do at various developmental levels (i.e., early elementary school, later elementary school, middle school, and high school).

Planning

The framework includes a planning section. It provides a model for using the standards and benchmarks to create a local district curriculum as part of continuous school improvement. It discusses the importance of involving representatives from all stakeholders in the curriculum development process. In addition, it emphasizes the need for alignment among all of the processes that comprise continuous school improvement and focuses attention on placing student achievement at the center of all decision-making. It emphasizes the need for continuity in a K-12 curriculum. Continuity is developed by clearly defining benchmarks that establish increasingly complex demonstrations of rigorous standards.

Teaching and Learning

The section on teaching and learning describes standards that are the foundation to successful learning in all content areas. The standards include deep knowledge, higher-order thinking, substantive conversation, and connections to the world beyond the classroom. It illustrates the standards through sample

teaching vignettes in each of the content areas. It discusses the importance of incorporating strategies for technology, connecting with the learner, interdisciplinary learning, and making school-to-work connections into the curriculum.

Assessment System

The framework contains a section on assessment which describes the need for developing a local assessment system to monitor student growth and program effectiveness. This section of the framework is divided into three parts. The first part provides a rationale for why an assessment system is needed. The second part describes how teachers can develop performance assessments based on the content standards and benchmarks. The third part discusses important issues related to building an assessment system that aligns local assessment practices with state assessment.

Professional Development

The section on professional development lists standards for the context, content, and process of professional development experiences. It includes a process for designing professional development which aligns with school improvement, curriculum content, student learning, and assessment needs. A vignette of one teacher's personal, professional development experiences is provided to illustrate Michigan's Standards for Professional Development.

Executive Summaries and Glossary

The appendices of the framework contain executive summaries of important resources that will aid a district as it develops, implements, and monitors its local curriculum. A glossary of framework terms is also provided.

Tier II

Toolkits

Tier II contains a collection of toolkits designed to help districts with specific tasks such as conducting discrepancy analyses. There are additional toolkits to guide districts in incorporating principles associated with connecting with the learner, technology, curriculum integration, and making school-to-work connections. There are toolkits on planning subject area instructional units, designing classroom assessments, and planning a district assessment system. *(Some of the above mentioned toolkits are still under development.)*

Tier III

Resources

Tier III contains content-area specific resources that help clarify the curriculum development process described in the framework. These include resources such as the *Science Education Guidebook*, the *Mathematics Teaching and Learning Sample Activities*, *Guidelines for the Professional Development of Teachers of English Language Arts*, and *Powerful & Authentic Social Studies Standards for Teaching.* It also

contains a guidebook written specifically for parents and the business community explaining the elements of the framework.

HOW TO USE THE FRAMEWORK

District school improvement committees and curriculum development committees will find the framework and its toolkits very useful as they begin the process of creating a standards-based curriculum. Reading and discussing the contents of the framework will help school improvement committee members gain a clearer understanding of the curriculum development process. The toolkits will help subcommittees develop techniques for creating and aligning curriculum, assessment, and instruction. They will also help districts make decisions about the professional development strategies which will most effectively help their students reach targeted achievement goals.

The first step in using the framework is to make sure that all interested parties are familiar with its content. Then an analysis to determine what needs to be done should be completed. Once the district identifies the tasks that need to be completed, a plan for structuring committees and a time-line for completing the tasks should be designed.

The framework is intended for use by all districts. While the writers used the structure of a middle-sized district as a frame of reference, the content and processes it describes are equally important for large and small districts. Although private schools are not bound by the core curriculum requirements of the Michigan School Code, they may find the framework useful as a tool for curriculum development. Large districts, small districts, private schools, and public school academies may choose to modify the process to reflect their organizational structures. The number and size of committees needed to implement the framework will vary from district to district, but the task will remain the same: to align curriculum, instruction, assessment, and professional development for the purpose of increasing student achievement of rigorous content standards.

Michigan Curriculum Framework

TIER I

This document introduces the framework standards and describes the components and processes needed to develop K-12 curricula.

- **Introduction**
- **Standards & Benchmarks**
- **Planning**
- **Teaching and Learning**
- **Assessment System**
- **Professional Development**
- **Executive Summaries of Toolkits and Other Resources**
- **Parent and Business Leader Guides (under development)**

TIER II

These documents are toolkits designed to help districts achieve alignment while developing curriculum, instruction, and assessment consistent with their standards and benchmarks. (For availability, see http://www.mde.state.mi.us)

Discrepancy Analysis

1. Analysis of Curriculum
2. Analysis of Instruction
3. Analysis of Assessment
4. Analysis of Professional Development
5. Analysis of School Operations

Connecting with the Learner
Technology
Curriculum Integration
Connecting School-To-Work

Local Assessment System

TIER III

These resources are specific to each content area and help clarify and strengthen the curriculum development processes described in the first two tiers. (For availability, see http:/www.mde.state.mi.us)

- Michigan Geography Framework Poster
- Analysis of Mathematics Instructional and Assessment Materials
- Mathematics Research Component
- Social Studies Curriculum Planning Guide

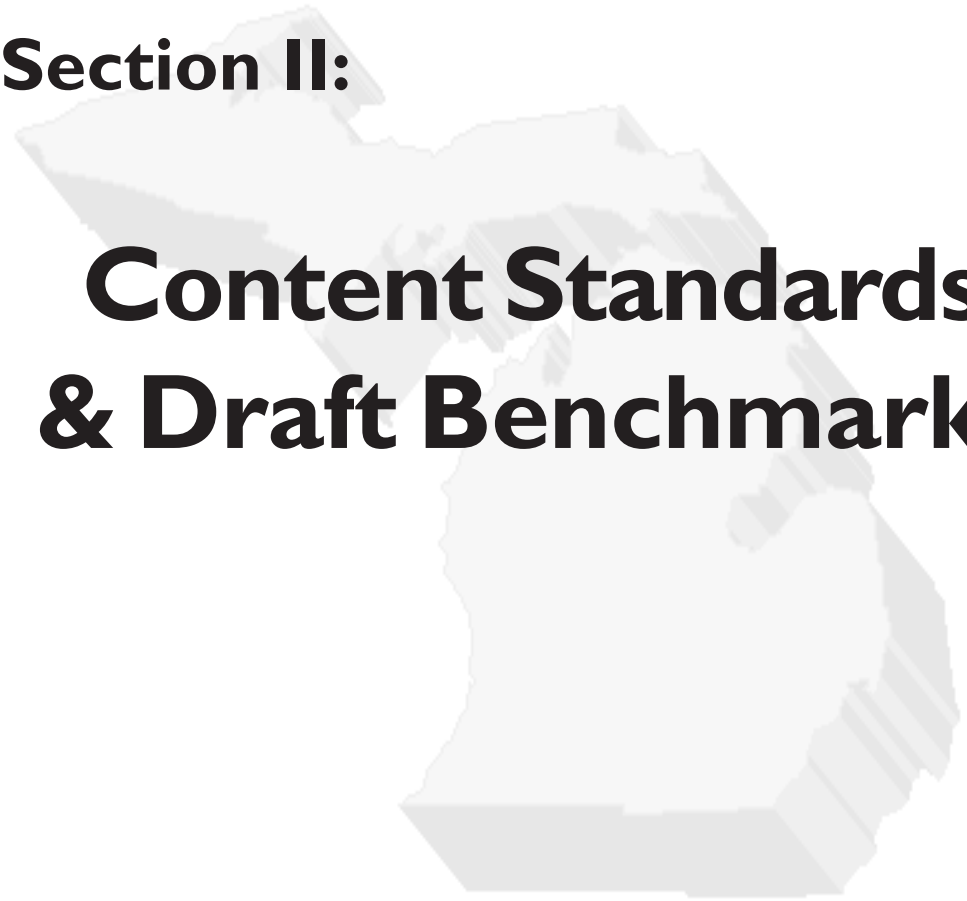
- Science Education Guidebook
- *New Directions* Science Teaching Units
- Profiles of Early Literacy Instruction in Primary Classrooms
- A Collection of English Language Arts Vignettes
- Readings from the Demonstration Projects
- Mathematics Teaching and Learning Sample Activities
- Standards Based Geography Units
- Powerful & Authentic Social Studies: Standards for Teaching and Learning

- Mathematics Assessment Framework
- The Model Assessment Items Resource Book *(Science)*
- Michigan Assessment Prototypes for Geography
- Plan for Statewide Assessment of Social Studies
- Social Studies Assessment Guide

- Guidelines for the Professional Development of Teachers of English Language Arts
- Mathematics Professional Development Component
- Powerful and Authentic Social Studies Professional Development Package
- Powerful and Authentic Social Studies Standards for Teaching

Section II:

Content Standards & Draft Benchmarks



Content Standards & Draft Benchmarks

At its July 19, 1995, meeting, the Michigan State Board of Education unanimously adopted the model content standards for curriculum. The content standards provide descriptions of what students should know and be able to do in the subject areas of English language arts, social studies, mathematics and science. In addition, benchmarks in each of the content areas were drafted to further clarify the content standards. The standards and benchmarks are not a state curriculum, but are specifically designed to be used by local districts as they develop their curricula.

The model content standards for curriculum and accompanying draft benchmarks will assist in the development of quality comprehensive local curricula, foster local diversity in establishing high quality learning expectations, and give parents, as customers within an education marketplace, an accountability tool. In addition, they will serve as a basis for revisions and new test development for the MEAP and High School Proficiency Tests. They will provide a common denominator to determine how well students are performing and will assure that all students are measured on the same knowledge and skills using the same method of assessment.

Model Content Standards for Curriculum

The model content standards for curriculum were revised, based upon public input, by writing teams in each of the content areas. The Curriculum Framework Joint Steering Committee which includes representatives from business, education, government, professional organizations, and labor was extensively consulted in the development and revision of the standards. The State Board also made revisions to the standards during its July 19, 1995, meeting.

Benchmarks

The draft benchmarks provide indicators of student expectations at various developmental levels including elementary, middle school, and high school. The working draft benchmarks are the most current versions and represent the efforts made by teams of subject area specialists with input from over 2,000 Michigan citizens.

Field reviews on the benchmarks were held to gather comments from teachers, parents, administrators, and community members. Additionally, the benchmarks have been reviewed for consistency with the model content standards for curriculum. The department continues the process of field testing the benchmarks at the Curriculum Framework projects' school demonstration sites.

SOC.II.1.LE.1

SOC. Social Studies *(Subject Area)*

II. Geographic Perspective *(Content Strand)*

1. All students will describe, compare, and explain the locations and characteristics of places, cultures, and settlements. (People, Places and Cultures) *(Content Standard)*

LE. Later Elementary

1. Locate and describe cultures and compare the similarities and differences among the roles of women, men and families. *(Later Elementary Benchmark)*

Because the benchmarks are continuously being revised to further clarify the standards and reflect the learning needs of Michigan's students, districts should consult electronic versions in order to ensure that they are working with the most current revisions. The model content standards for curriculum and the working draft benchmarks are available through the Internet on the Department of Education's gopher server (gopher://gopher.mde.state.mi.us), and through the World Wide Web (http://cdp.mde.state.mi.us).

Please note: The standards and benchmarks have been coded so that districts can more easily refer to them in their curriculum, instruction, assessment, and professional development activities. The numbering system will be useful as districts conduct discrepancy analyses as part of the continuous school improvement decision-making process. One system for numbering has been applied to all of the content standards and benchmarks in an attempt to provide consistency and facilitate curriculum alignment.

The numbering system begins with the subject area. English language arts is assigned the code of ELA; Mathematics, MAT; Social Studies, SOC; and Science, SCI. The first numeral in the code is a Roman numeral; it identifies the content area strand. The second numeral is an Arabic numeral; it identifies a content standard. The letters that follow the content standard signify cluster levels such as: E (elementary), EE (early elementary), LE (later elementary), MS (middle school), and HS (high school). The third numeral is another Arabic numeral; it identifies a benchmark.

The coding system has been used to identify standards and benchmarks in the sections on assessment and teaching and learning. Please note that although one coding system is used, each set of content areas and benchmarks has some unique characteristics. For instance, the English language arts standards and benchmarks do not identify strands. It is very important to study the standards and benchmarks carefully so they can be used to their full advantage.

Mathematics

Vision Statement

Mathematics is the science of patterns and relationships. It is the language and logic of our technological world. Mathematical power is the ability to explore, to conjecture, to reason logically and to use a variety of mathematical methods effectively to solve problems. The ultimate goal of mathematics education is for all students to develop mathematical power to participate fully as a citizen and worker in our contemporary world.

A mathematically powerful individual should be able to:

- reason mathematically;
- communicate mathematically;
- problem solve using mathematics; and,
- make connections within mathematics and between mathematics and other fields.

The fifteen content standards have been categorized into the following six strands:

I. **Patterns, Relationships, and Functions**

II. **Geometry and Measurement**

III. **Data Analysis and Statistics**

IV. **Number Sense and Numeration**

V. **Numerical and Algebraic Operations and Analytical Thinking**

VI. **Probability and Discrete Mathematics**

OVERVIEW OF THE MATHEMATICS CONTENT STRANDS

Strand I. Patterns, Relationships, and Functions

Patterns, relationships and functions comprise one of the most important themes in the study of mathematics. Mathematical thinking begins with the recognition of similarities among objects or events, proceeds to generalization and abstraction, and culminates in the ability to understand, explain and make predictions. Contexts that exhibit structure and regularity provide rich opportunities for describing the physical world, studying mathematics and solving problems.

Standard I.1 Patterns
Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.

Wherever there is mathematics there are patterns, and wherever there are patterns there is mathematics. Patterns are regularities or similarities that characterize sets of numbers, shapes, graphs, tables or other mathematical objects. Mathematicians look for patterns in everything they do; thus, mathematics is frequently defined as the science of patterns. In studying mathematics, students learn to recognize, describe, analyze and create patterns, to extend and generalize patterns, to create mathematical models based on observed patterns, and to predict the behavior of real-world phenomena based on such observed patterns. They learn to communicate the nature of mathematical patterns and relationships in various ways including words, physical models, diagrams, tables, charts, graphs, and equations. Since each representation highlights different aspects of the patterns and relationships, students must be able to construct multiple representations of mathematical relationships and to translate among them.

Standard I.2 Variability and Change
Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

Variability and change are as fundamental to mathematics as they are to the physical world, and an understanding of the concept of a variable is essential to mathematical thinking. Students must be able to describe the relationships among variables, to predict what will happen to one variable as another variable is changed, and to compare different patterns of change. The study of variability and change provides a basis for making sense of the world and of mathematical ideas.

Strand II. Geometry and Measurement
We live in a three-dimensional world. In order to interpret and make sense of that world, students need both analytical and spatial abilities. Geometry and measurement, which involve notions of shape, size, position, and dimension, are used extensively to describe and understand the world around us.

Standard II.1 Shape and Shape Relationships
Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.

Spatial sense is developed when students recognize, draw, construct, visualize, compare, classify and transform geometric shapes in both two and three dimensions. They learn to identify those characteristics that are necessary to define a given shape, and they can differentiate one shape from another. Students also develop an awareness of the properties of a shape and of the relationships among shapes. This

includes hierarchical classifications of shapes (e.g., all squares are rhombuses), relationships among components of a shape (e.g., opposite sides of a rectangle are parallel), symmetries of a shape, congruence and similarity.

Standard II.2 Position
Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.

Position refers to the location of physical objects or points in space as well as to the relative locations and positions of objects, points, lines, planes and other geometric elements. It includes such notions as betweenness, collinearity and coordinates in two and three dimensions, as well as the locus of a point as it moves through space and the location of special points.

Standard II.3 Measurement
Students compare attributes of two objects or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

Measurement reflects the usefulness and practicality of mathematics and puts students in touch with the world around them. Measurement requires the comparison of an attribute (distance, surface, capacity, mass, time, temperature) between two objects or to a known standard, the assignment of a number to represent the comparison, and the interpretation of the results. Measurement also introduces students to the important concepts of precision, approximation, tolerance, error and dimension.

Strand III. Data Analysis and Statistics

We live in a sea of information. In order not to drown in the data that inundate our lives every day, we must be able to process and transform data into useful knowledge. The ability to interpret data and to make predictions and decisions based on data is an essential basic skill for every individual.

Standard III.1 Collection, Organization and Presentation of Data
Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.

Knowing what data to collect and where and how to collect them is the starting point of quantitative literacy. The mathematics curriculum should capitalize on students' natural curiosity about themselves and their surroundings to motivate them to collect and explore interesting statistics and measurements derived from both real and simulated situations. Once the data are gathered, they must be organized into a useful form, including tables, graphs, charts and pictorial representations. Since different representations highlight different patterns within the data, students should develop skill in representing and reading data displayed in different

formats, and they should discern when one particular representation is more desirable than another.

Standard III.2 Description and Interpretation
Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.

Students must be able to examine data and describe salient characteristics of the distribution. They also must be able to relate the data to the physical situation from which they arose. Students should use the data to answer key questions and to convince and persuade.

Standard III.3 Inference and Prediction
Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Based on known data, students should be able to draw defensible inferences about unknown outcomes. They should be able to make predictions and to identify the degree of confidence that they place in their predictions.

Strand IV. Number Sense and Numeration

Number sense is to mathematics what vocabulary is to language. Students must learn to quantify and measure, concretely at first and increasingly more abstractly as they mature. They also must develop an understanding of numeration systems and of the structure of such systems. They must learn to estimate mathematical quantities and to represent and communicate mathematical ideas in the language of mathematics.

Standard IV.1 Concepts and Properties of Numbers
Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.

Fundamental questions like "What is a number?" or "What is three?" can be deceptively difficult to answer. Students require extensive involvement with concrete experiences of counting and measuring in order to develop an intuitive sense about number. Through both informal and formal means, students develop understanding about important properties of numbers such as even vs. odd, whole number vs. fraction, positive vs. negative. They understand the existence of different sets of numbers (whole numbers, integers, rationals, reals, ...) and the properties of special numbers such as 0, 1, π, or the inverse of a number.

Standard IV.2 Representation and Uses of Numbers
Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Students recognize that numbers are used in different ways such as to answer the questions "How many?" (counting), "How much?" (measuring), and "Which one?" (ordering). They understand that a numerical quantity can be represented in many different ways, and they can produce multiple representations of numbers (e.g., fractions, decimals, and percents in the middle grades; vectors and coordinate representations in later years) and they can translate easily among equivalent representations. As students mature from the middle school on, they develop a solid understanding of both linearity and proportionality.

Standard IV.3 Number Relationships
Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Students develop understanding of important relationships among numbers including the relationships of (=, ≠) and (<, >); of opposites (additive inverses) and reciprocals (multiplicative inverses); of factors and multiples; of primes, composites, and relatively prime numbers; of powers and roots. They understand and can represent very large and very small numbers and can compare the orders of magnitude of numbers.

Strand V. Numerical and Algebraic Operations and Analytical Thinking

The ability to represent quantitative situations with algebraic symbolism, numerical operations and algebraic thinking is essential to solving problems in significant contexts and applications. The concepts of number and variable and their symbolic representation and manipulation are central to the understanding of arithmetic and its generalization in algebra. The contemporary applications of mathematics in virtually every field of work and study rely on algebraic and analytic thinking and communication as fundamental tools.

Standard V.1 Operations and Their Properties
Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

The ultimate reason for mastering the operations of arithmetic and algebra is to solve problems. To that end, understanding the basic computational operations and their algorithms is essential for competence in mathematics, but the emphasis must be on understanding and using the operations, not on memorizing algorithms. In computation, understanding and accuracy are always more important than speed. Understanding the operations requires the concomitant understanding and application of the properties of those operations, and it involves knowing what operations to use in a particular situation. There is no one way to perform a calculation. Students must be competent in performing calculations, but they need not have a rigid adherence to one algorithm. Methods of computation include proficiency with mental calculation, paper and pencil, and calculators; the

ability to represent computations with manipulatives and geometric models; and the discernment of which computational method to use in a given situation. Computational methods also involve estimating and assessing the reasonableness of the results of a computation.

Standard V.2 Algebraic and Analytic Thinking
Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Mathematical representations allow us to visualize and understand problems. These representations may be numerical, literal, symbolic, graphical, pictorial or physical. Facility with multiple representations of numerical and algebraic concepts and relationships is essential to mathematical competence. This includes the development of "symbol sense" as well as "number sense" and the understanding that the notion of solution involves a process as well as a product. Thus, the solution of a mathematical problem requires both an understanding of the question for which an answer is sought and the development of a strategy to obtain that answer. The context of the problem determines the nature and the degree of precision of the required solution. The increasing use of quantitative methods in all disciplines has made algebra the fundamental tool for mathematical applications. Algebraic thinking is learned most effectively when it is studied in the context of applications, both mathematical and real-world, that reveal the power of algebra to model real problems and to generalize to new situations. Students should use algebraic techniques to analyze and describe relationships, to model problem situations, and to examine the structure of mathematical relationships. The algebra curriculum should employ contemporary technology, including spreadsheets and graphical analysis, to emphasize conceptual understanding of algebra and analytic thinking as sophisticated means of representation and as powerful problem-solving tools.

Strand VI. Probability and Discrete Mathematics

Contemporary uses of mathematics demand that students learn to deal with uncertainty, to make informed decisions based on evidence and expectations, to exercise critical judgment about conclusions drawn from data, and to apply mathematical models to real-world phenomena. The technological world in which we live also depends upon information and the communication of information and upon applications of systems with separate (discrete) entities. Topics of discrete mathematics such as counting and permutation problems, matrix operations, vertex-edge networks, and relationships among finite sets have significant real-world applications that students will encounter in diverse fields of work and study.

Standard VI.1 Probability
Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that

can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.

Dealing with uncertainty and making predictions and decisions in the face of uncertainty are essential skills for coping with the modern world. Students must develop an understanding of the notion of uncertainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available. They also must be able to make critical judgments about claims that are made in probabilistic situations.

Standard VI.2 Discrete Mathematics
Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.

Discrete (discontinuous) mathematics has grown in significance in recent years and today has applications in many important practical situations such as scheduling, routing, sequencing, networking, organizing and classifying. Important ideas like recurrence relations, induction and algorithm design also have practical applications in a variety of fields. Computers, which are finite, discrete machines, require an understanding of discrete mathematics for the solution of problems using computer methods.

MATHEMATICS

CONTENT STANDARDS AND WORKING DRAFT BENCHMARKS

I. Patterns, Relationships and Functions

Content Standard 1: Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

Elementary	Middle School	High School
1. Recognize, describe and extend numerical and geometric patterns.	1. Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.	1. Analyze and generalize mathematical patterns including sequences, series and recursive patterns.
2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.	2. Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.	2. Analyze, interpret and translate among representations of patterns including tables, charts, graphs, matrices and vectors.
3. Use patterns to describe real-world phenomena.	3. Use patterns and their generalizations to make and justify inferences and predictions.	3. Study and employ mathematical models of patterns to make inferences, predictions and decisions.
4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).	4. Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns and symmetric and spatial patterns.	4. Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations or relations.
5. Apply their experiences with patterns to help solve problems and explore new content.	5. Use patterns and generalizations to solve problems and explore new content.	5. Use patterns and reasoning to solve problems and explore new content.

Content Standard 2: Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

Elementary	Middle School	High School
1. Recognize change and variability when it occurs in a variety of settings.	1. Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change and patterns of variation.	1. Identify and describe the nature of change and begin to use the more formal language such as rate of change, continuity, limit, distribution and deviation.
2. Recognize that change is often predictable, but variable, and that patterns emerge that help to describe the change.	2. Connect an initial state to a final state and generalize a rule that describes a pattern of change.	2. Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).

3. Explore change, and realize that changes are frequently interdependent.	3. Begin to investigate applications in bivariate data and linear relationships and explore questions of what will happen to one quantity if another variable is changed.	3. Expand their understanding of function to include non-linear functions, composition of functions, inverses of functions, and piecewise- and recursively-defined functions.
4. Use tables, charts, open sentences and hands-on models to represent change and variability.	4. Represent variability or change by ordered pairs, tables, graphs and equations.	4. Represent functions using symbolism such as matrices, vectors and functional representation (f(x)).
5. Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.	5. Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.	5. Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular and trigonometric functions, and realize that many different situations can be modeled by a particular type of function.
6. Explore variability and change in a variety of contexts, investigations and problems.	6. Continue to explore relationships arising from interesting contexts and use variables and relationships to solve mathematical problems.	6. Increase their use of functions and mathematical models to solve problems in context.

II. Geometry and Measurement

Content Standard 1: Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes. (Shape and Shape Relationships)

Elementary	Middle School	High School
1. Recognize and name familiar shapes in one, two and three dimensions such as lines, rectangles and spheres and informally discuss the shape of a graph.	1. Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.	1. Use shape to identify plane and solid figures, graphs, loci, functions and data distributions.
2. Describe the attributes of familiar shapes.	2. Generalize the characteristics of shapes and apply their generalizations to classes of shapes.	2. Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes.
3. Compare, sort and classify familiar shapes.	3. Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.	3. Use transformational, coordinate or synthetic methods to verify (prove) the generalizations they have made about properties of classes of shapes.
4. Draw and build familiar shapes.	4. Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.	4. Draw and construct shapes in two and three dimensions and analyze and justify the steps of their constructions.

5. Explore ways to combine, dissect and transform shapes.	5. Combine, dissect and transform shapes.	5. Study transformations of shapes using isometries, size transformations and coordinate mappings.
6. Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.	6. Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.	6. Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity and incidence.
7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

Content Standard 2: Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

Elementary	Middle School	High School
1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between and locations on the number line, on a coordinate graph and on a map.	1. Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude and midpoints.	1. Locate and describe objects in terms of their position, including polar coordinates, three-dimensional Cartesian coordinates, vectors and limits.
2. Locate and describe objects in terms of their orientation, direction and relative position, including up, down, front, back, N- S- E- W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.	2. Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N- S- E- W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.	2. Locate and describe objects in terms of their orientation and relative position, including displacement (vectors), phase shift, maxima, minima and inflection points; give precise mathematical descriptions of symmetries.
3. Explore what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging or reducing it.	3. Describe translations, reflections, rotations and dilations using the language of transformations and employ transformations to verify congruence of figures.	3. Give precise mathematical descriptions of transformations and describe the effects of transformations on size, shape, position and orientation.
	4. Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.	4. Describe the locus of a point by a rule or mathematical expression; trace the locus of a moving point.
5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.	5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.	5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.

Content Standard 3: Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision. (Measurement)

Elementary	Middle School	High School
1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.	1. Select and use appropriate tools; measure objects using standard units in both the metric and common systems and measure angles in degrees.	1. Select and use appropriate tools; make accurate measurements using both metric and common units, and measure angles in degrees and radians.
2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), area, perimeter, capacity, time, temperature and money.	2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume and angle.	2. Continue to make and apply measurements of length, mass (weight), time, temperature, area, volume, angle; classify objects according to their dimensions.
3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is "a good estimate."	3. Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is "close enough."	3. Estimate measures with a specified degree of accuracy and evaluate measurements for accuracy, precision and tolerance.
4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.	4. Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area) but not necessarily on another (e.g., perimeter).	4. Interpret measurements and explain how changes in one measure may affect other measures.
5. Explore scale drawings, models and maps and relate them to measurements of real objects.	5. Use proportional reasoning and indirect measurements to draw inferences.	5. Use proportional reasoning and indirect measurements, including applications of trigonometric ratios, to measure inaccessible distances and to determine derived measures such as density.
6. Apply measurement to describe the real world and to solve problems.	6. Apply measurement to describe the real world and to solve problems.	6. Apply measurement to describe the real world and to solve problems.

III. Data Analysis and Statistics

Content Standard 1: Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

Elementary	Middle School	High School
1. Collect and explore data through counting, measuring and conducting surveys and experiments.	1. Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.	1. Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.
2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.	2. Organize data using tables, charts, graphs, spreadsheets and data bases.	2 Organize data using tables, charts, graphs, spreadsheets and data bases.

3. Present data using a variety of appropriate representations and explain the meaning of the data.	3. Present data using a variety of appropriate representations and explain why one representation is preferred over another or how a particular representation may bias the presentation.	3. Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.
4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.	4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.	4. Identify what data are needed to answer a particular question or solve a given problem and design and implement strategies to obtain, organize and present those data.

Content Standard 2: Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively. (Description and Interpretation)

Elementary	Middle School	High School
1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.	1. Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.	1. Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.
2. Describe the shape of the data using informal language.	2. Describe the shape of a data distribution and identify the center, the spread, correlations and any outliers.	2. Describe the shape of a data distribution and determine measures of central tendency, variability and correlation.
3. Draw, explain and justify conclusions, such as trends based on data.	3. Draw, explain and justify conclusions based on data.	3. Use the data and their characteristics to draw and support conclusions.
4. Raise and answer questions about the source, collection, organization and presentation of data, as well as the conclusions drawn from the data; explore biases in the data.	4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.	4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the sources of bias and measures taken to eliminate such bias.
5. Formulate questions and problems and gather and interpret data to answer those questions.	5. Formulate questions and problems and gather and interpret data to answer those questions.	5. Formulate questions and problems and gather and interpret data to answer those questions.

Content Standard 3: Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions. (Inference and Prediction)

Elementary	Middle School	High School
1. Make and test hypotheses.	1. Make and test hypotheses.	1. Make and test hypotheses.
2. Conduct surveys, samplings and experiments to solve problems and answer questions of interest to them.	2. Design experiments to model and solve problems using sampling, simulations and controlled investigations.	2. Design investigations to model and solve problems; also employ confidence intervals and curve fitting in analyzing the data.

3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.
4. Make and explain predictions based on data.	4. Make predictions and decisions based on data, including interpolations and extrapolations.	4. Make predictions and decisions based on data, including interpolations and extrapolations.
5. Make predictions to answer questions and solve problems.	5. Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.	5. Employ investigations, mathematical models, and simulations to make inferences and predictions to answer questions and solve problems.

IV. Number Sense and Numeration

Content Standard 1: Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers. (Concepts and Properties of Numbers)

Elementary	Middle School	High School
1. Develop an understanding of whole numbers and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.	1. Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.	1. Develop an understanding of irrational, real and complex numbers.
2. Investigate and develop an understanding of the base-10 place-value system.	2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.	2. Use the (a+bi) and polar forms of complex numbers.
3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.	3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and π, and the additive and multiplicative inverses.	3. Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including π, i, e, and conjugates.
4. Apply their understanding of number systems to model and solve problems.	4. Apply their understanding of number systems to model and solve mathematical and applied problems.	4. Apply their understanding of number systems to model, and solve mathematical and applied problems.

Content Standard 2: Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations. (Representation and Uses of Numbers)

Elementary	Middle School	High School
1. Represent whole numbers, fractions and decimals using concrete, pictorial and symbolic representations.	1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.	1. Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.

2. Explore and recognize different representations for the same number and explain why they are the same.	2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.	2. Develop an understanding of more complex representations of numbers, including exponential and logarithmic expressions, and select an appropriate representation to facilitate problem solving.
3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).	3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.	3. Determine when to use rational approximations and the exact values of numbers such as e, π and the irrational.
4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.	4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.	4. Apply estimation in increasingly complex situations.
5. Select appropriate numbers and representations in order to solve problems.	5. Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.	5. Select appropriate representations for numbers, including representations of rational and irrational numbers and coordinate and vector representations of complex numbers, in order to simplify and solve problems.

Content Standard 3: Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers. (Number Relationships)

Elementary	Middle School	High School
1. Compare and order numbers using "equal," "less than" or "greater than."	1. Compare and order integers and rational numbers using relations of equality and inequality.	1. Compare and order real numbers and compare rational approximations to exact values.
2. Use part-whole relationships to explore numbers, develop number concepts and understand computation.	2. Express numerical comparisons as ratios and rates.	2. Express numerical comparisons as ratios and rates.
3. Classify numbers as even or odd and explore concepts of factors and multiples.	3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.	3. Extend the relationships of primes, factors, multiples and divisibility in an algebraic setting.
	4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.	4. Express number relationships using positive and negative rational exponents, logarithms and radicals.
5. Apply their understanding of number relationships in solving problems.	5. Apply their understanding of number relationships in solving problems.	5. Apply their understanding of number relationships in solving problems.

V. Numerical and Algebraic Operations and Analytical Thinking

Content Standard 1: Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties).

Elementary	Middle School	High School
1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.	1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.	1. Present and explain geometric and symbolic models for operations with real and complex numbers and algebraic expressions.
2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.	2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.	2. Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology and, for simple instances, with paper-and-pencil algorithms.
3. Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.	3. Describe the properties of operations with rationals and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.	3. Describe the properties of operations with numbers, algebraic expressions, vectors and matrices, and make generalizations about the properties of given mathematical systems.
4. Apply operations efficiently and accurately in solving problems.	4. Efficiently and accurately apply operations with integers, rational numbers and simple algebraic expressions in solving problems.	4. Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices and vectors in solving problems.

Content Standard 2: Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

Elementary	Middle School	High School
1. Write and solve open sentences (e.g., $\Diamond + \Delta = 5$) and write stories to fit the open sentence.	1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.	1. Identify important variables in a context, symbolize them and express their relationships algebraically.
2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.	2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.	2. Represent algebraic concepts and relationships with matrices, spreadsheets, diagrams, graphs, tables, physical models, vectors, equations and inequalities; and translate among the various representations.

3. Find replacements for the variable(s) in open sentences.	3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.	3. Solve linear equations and inequalities algebraically and non-linear equations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.
4. Use analytic thinking to describe situations and solve problems.	4. Analyze problems modeled by linear functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.	4. Analyze problems that can be modeled by functions, determine strategies for solving the problems and evaluate the adequacy of the solutions in the context of the problems.
	5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.	5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

VI. Probability and Discrete Mathematics

Content Standard 1: Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations. (Probability)

Elementary	Middle School	High School
1. Explain the difference between chance and certainty and give examples to illustrate their understanding.	1. Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.	1. Develop an understanding of randomness and chance variation and describe chance and certainty in the language of probability.
2. Compare events and describe them as “more likely” or “less likely” and use the language of fractions to describe simple probabilities.	2. Describe probability as a measure of certainty ranging from 0 to 1 and conduct activities that allow them to express probabilities of simple events in mathematical terms.	2. Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.
3. Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.	3. Conduct experiments and give examples to illustrate the difference between dependent and independent events.	3. Analyze events to determine their dependence or independence and calculate probabilities of compound events.
4. Conduct experiments, record the outcomes, examine those outcomes to determine if they make sense and search for explanations of the outcomes.	4. Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathematically (theoretical), and explain how the empirical probability changes for a large number of trials.	4. Use sampling and simulations to determine empirical probabilities and, when appropriate, compare them to the corresponding theoretical probabilities; understand and apply the law of large numbers.

5. Conduct probability experiments and simulations to model and solve problems.	5. Conduct probability experiments and simulations to model and solve problems.	5. Conduct probability experiments and simulations, to model and solve problems, including compound events.

Content Standard 2: Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design. (Discrete Mathematics)

Elementary	Middle School	High School
1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.	1. Use manipulatives, diagrams and the fundamental theorem of counting to count permutations and combinations.	1. Derive and use formulas for calculating permutations and combinations.
2. Explore sets and set relationships by sorting and classifying objects.	2. Use sets and set relationships to explore and solve simple algebraic and geometric problems.	2. Use sets and set relationships to represent algebraic and geometric concepts.
3. Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.	3. Solve problems involving networks, for example planning delivery routes or counting paths between points.	3. Use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees and adjacency matrices.
4. Explore now-next patterns.	4. Explore recurrence relations and iterations.	4. Analyze and use discrete ideas, such as induction, iteration and recurrence relations.
5. Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.	5. Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.	5. Describe and analyze efficient algorithms to accomplish a task or solve a problem in a variety of contexts, including practical, mathematical and computer-related situations.
6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).	6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).	6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems) and decide upon a best solution (optimization problems).