ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
Εθνικόν και Καποδιστριακόν
Πανεπιστήμιον Αθηνών

Ιστορία νεότερων Μαθηματικών

Ενότητα 3: Η Άλγεβρα της Αναγέννησης

Παπασταυρίδης Σταύρος
Σχολή Θετικών Επιστημών
Τμήμα Μαθηματικών

Περιγραφή Ενότητας

Ιταλοί Αβακιστές. Αλγεβρικός Συμβολισμός. Άλγεβρα στην Γαλλία, Γερμανία, Αγγλία. Εξισώσεις τρίτου και τετάρτου βαθμού. Μιγαδικοί αριθμοί. Εξισώσεις τετάρτου βαθμού και συμμετρίες.

Περιεχόμενα Υποενότητας

- Rafael Bombelli
- Άλγεβρα, 1572. Δυνάμεις, μιγαδικοί, πράξεις μιγαδικών
- Η «Άλγεβρα» του Bombelli. Αναφορά σε Διόφαντο, πολλαπλασιαμός ριζικών, πραγματικών, μιγαδικών. Συμβολισμοί

Η Άλγεβρα της Αναγέννησης

Bombelli και μιγαδικοί αριθμοί

Rafael Bombelli (1526-1572) (1/4)

- Rafael Bombelli (baptised on 20 January 1526; died 1572[a]) was an Italian mathematician.
- Born in Bologna, he is the author of a treatise on algebra and is a central figure in the understanding of imaginary numbers.
- He was the one who finally managed to address the problem with imaginary numbers. In his 1572 book, L'Algebra, Bombelli solved equations using the method of del Ferro/Tartaglia. He introduced the rhetoric that preceded the representative symbols +i and -i and described how they both worked.

Rafael Bombelli (1526-1572) (2/4)

- Rafael Bombelli was baptised on 20 January 1526[1] in Bologna, **Papal States**. He was born to **Antonio Mazzoli**, a wool merchant, and Diamante Scudieri, a tailor's daughter. The Mazzoli family was once quite powerful in Bologna. When Pope Julius II came to power, in 1506, he exiled the ruling family, the Bentivoglios. The Bentivoglio family attempted to retake Bologna in 1508, but failed. Rafael's grandfather participated in the coup attempt, and was captured and executed.

Rafael Bombelli (1526-1572) (3/4)

- Later, Antonio was able to return to Bologna, having changed his surname to Bombelli to escape the reputation of the Mazzoli family. Rafael was the oldest of six children. **Rafael received no college education, but was instead taught by an engineer-architect by the name of Pier Francesco Clementi**.
- Katz 3rd ed p. 405. Bombelli was educated as an engineer and spent much of his adult life working on engineering projects in the service of his patron, a Roman nobleman who was a favorite of Pope
- Paul III. The largest project in which he was involved was the reclamation of the marshes in the Val di Chiana into arable
- land. Today that valley, extending southeast for about sixty miles between the Arno and the Tiber, is still one of the most fertile in central Italy.

Rafael Bombelli (1526-1572) (4/4)

- Bombelli later served as a consultant on a proposed project for the draining of the Pontine Marshes near Rome.
- During a lull in the reclamation work caused by a war in the area, he was able to work on his algebra treatise at his patron’s villa in Rome sometime between 1557 and 1560.
- Other professional engagements delayed the printing of it, however, and it did not appear until shortly before his death in 1572.
- Rafael Bombelli felt that none of the works on algebra by the leading mathematicians of his day provided a careful and thorough exposition of the subject. Instead of another convoluted treatise that only mathematicians could comprehend, Rafael decided to write a book on algebra that could be understood by anyone. His text would be self-contained and easily read by those without higher education.
- Σχόλιο: την έγραψε στα ιταλικά.

Άλγεβρα (1572)

- Bombelli's *Algebra was more in the tradition of Luca Pacioli's Summa and the German Coss works than was Cardano's book. Bombelli began the book with elementary material* and gradually worked up to the solving of cubic and quartic equations.
- Like Cardano, he gave a separate treatment to each class of cubics, but he expanded on Cardano's brief treatment of quartics by giving a separate section to each of those classes as well.
- After dealing with the theoretical material, he presented the student with a multitude of problems using the techniques developed in the earlier chapters. He had originally intended to include practical problems similar to those of the earlier abacus works, but after studying a copy of
- Diophantus's *Arithmetica at the Vatican Library, he decided to replace these with abstract* numerical problems taken from Diophantus and other sources.

Άλγεβρα (1572): Δυνάμεις

Recall that algebraic symbolism was gradually replacing the strictly verbal accounts of the Moslems and of the earliest Italian algebraists. Cardano had used some symbolism, but Bombelli's was a bit different. For example, he used $R.q.$ to denote the square root, $R.c.$ to denote the cube root, and similar expressions to denote higher roots. He used $\lfloor\ \rfloor$ as parentheses to enclose long expressions, as in $R.c.\lfloor 2\ p\ R.q.21\rfloor$, but kept the standard Italian abbreviations of p for plus and m for minus. His major notational innovation was the use of a semicircle around a number n to denote the nth power of the unknown. Thus, $x^3 + 6x^2 - 3x$ would be written as

$$1^{\overset{3}{\smile}}p\ 6^{\overset{2}{\smile}}m\ 3^{\overset{1}{\smile}}.$$

Άλγεβρα (1572). Μιγαδικοί (1/2)

- Writing powers numerically rather than in the German form of symbols allowed him easily to express the exponential laws for multiplying and dividing monomials.
- Late in the first part of the *Algebra, Bombelli introduced "another sort of cube root much* different from the former, which comes from the chapter on the cube equal to the thing and number; . . . this sort of root has it own algorithms for various operations and a new name."

name."[31] This root is the one that occurs in the cubic equations of the form $x^3 = cx + d$ when $(\frac{d}{2})^2 - (\frac{c}{3})^3$ is negative. Bombelli proposed a new name for these numbers, which are neither positive (*più*) nor negative (*meno*), that is, the modern imaginary numbers. The numbers written today as bi, $-bi$, respectively, Bombelli called *più di meno* (plus of minus) and *meno di meno* (minus of minus). For example, he wrote $2 + 3i$ as 2 *p di m* 3 and $2 - 3i$

Άλγεβρα (1572). Μιγαδικοί (2/2)

- As 2mdi m3. Bombelli presented the various laws of multiplication for these new (complex) numbers, such as
- pi`u di meno times pi`u di meno gives meno and pi`u di meno times meno di meno gives pi`u

$$((bi)(ci) = -bc, bi(-ci) \ = \ bc).$$

- Κατά κάποιο τρόπο συνεβόλιζε το σημερνό i, me “p di m”
- Σχόλιο. Imaginary numbers: ατυχής ονομασία !

Άλγεβρα (1572). Πράξεις μιγαδικών. “Sofistry”. (1/2)

To illustrate his rules, Bombelli gave numerous examples of the four arithmetic operations on these new numbers. Thus, to find the product of $\sqrt[3]{2+\sqrt{-3}}$ and $\sqrt[3]{2+\sqrt{-3}}$ one first multiplies $\sqrt{-3}$ by itself to get -3, then 2 by itself to get 4, then adds these two to get 1 for the “real” part. Next, one multiplies 2 by $\sqrt{-3}$ and doubles the result to get $\sqrt{-48}$. The answer is $\sqrt[3]{1+\sqrt{-48}}$. To divide 1000 by $2+11i$, Bombelli multiplied both numbers by $2-11i$. He then divided the new denominator, 125, into 1000, giving 8, which in turn he multiplied by $2-11i$ to get $16-88i$ as the result. Bombelli, although he noted that “the whole matter seems to rest on sophistry rather than on truth,”[22] nevertheless presented here for the first

Άλγεβρα (1572). Πράξεις μιγαδικών. “Sofistry”. (2/2)

seems to rest on sophistry rather than on truth,”[22] nevertheless presented here for the first time the rules of operation for complex numbers. It seems clear from his discussion that he developed these rules strictly by analogy to the known rules for dealing with real numbers. Arguing by analogy is a common method of making mathematical progress, even if one is not able to give rigorous proofs. Of course, because Bombelli did not know what these numbers “really” were, he could give no such proofs.

Επεξηγήσεις (1/5)

- Πιο συγκεκριμένα χρησιμοποιεί: $a^{1/3}b^{1/3} = (ab)^{1/3}$,
- $(a^{1/2})^2 = a$ και όχι $a^{1/2}a^{1/2} = (aa)^{1/2}$
- Question: Can we define f a complex function on C, so as to be continuous and $(f(z))2 = z$?
- Question: Can we define f a complex function on C, so as to be $(f(z))2 = z$ and
- $f(z_1)f(z_2) = f(z_1z_2)$
- Let $f(1) = 1$ (we should examine the case -1 too).
- Let $f(-1) = i$ (we should examine the case -i too).
- $f(1) = f(\,(-1)(-1)\,) = f(-1)f(-1) = ii = -1$

Επεξηγήσεις (2/5)

Proofs notwithstanding, with the rules for dealing with complex numbers now available, Bombelli could discuss how to use Cardano's formula for the case $x^3 = cx + d$ whether $(\frac{d}{2})^2 - (\frac{c}{3})^3$ is positive or negative. He first considered the example $x^3 = 6x + 40$. Cardano's procedure gives $x = \sqrt[3]{20 + \sqrt{392}} + \sqrt[3]{20 - \sqrt{392}}$, even though it is obvious that the answer is $x = 4$. Bombelli showed how one can see that the sum of the two cube roots is in fact 4. He assumed that $20 + \sqrt{392}$ equals the cube of a quantity of the form $a + \sqrt{b}$ for some numbers a and b, or $\sqrt[3]{20 + \sqrt{392}} = a + \sqrt{b}$. This implies that $\sqrt[3]{20 - \sqrt{392}} = a - \sqrt{b}$.

Επεξηγήσεις (3/5)

Multiplying these two equations together gives $\sqrt[3]{8} = a^2 - b$ or $a^2 - b = 2$. Furthermore, cubing the first equation and equating the parts without square roots gives $a^3 + 3ab = 20$. Bombelli did not attempt to solve this system of two equations in two unknowns by a general argument. Rather, he noted that the only possible integral value for a is $a = 2$. Fortunately, $b = 2$ then provides the other value in each equation, so Bombelli had shown that $\sqrt[3]{20 + \sqrt{392}} = 2 + \sqrt{2}$ and $\sqrt[3]{20 - \sqrt{392}} = 2 - \sqrt{2}$. It follows that the solution to the cubic equation is $x = (2 + \sqrt{2}) + (2 - \sqrt{2}) = 4$ as desired.

Επεξηγήσεις (4/5)

For the equation $x^3 = 15x + 4$, the Cardano formula gives

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}},$$

although again it is clear that the answer is $x = 4$. Bombelli used his newfound knowledge of complex numbers to apply the same method as above. He first assumed that $\sqrt[3]{2 + \sqrt{-121}} = a + \sqrt{-b}$. Then $\sqrt[3]{2 - \sqrt{-121}} = a - \sqrt{-b}$, and a short calculation leads to the two equations $a^2 + b = 5$ and $a^3 - 3ab = 2$. Again, Bombelli carefully showed that $a = 2$ was the only possibility. Then $b = 1$ provides the other solution and the desired cube root is $2 + \sqrt{-1}$. It follows that the solution to the cubic equation is $x = (2 + \sqrt{-1}) + (2 - \sqrt{-1})$ or $x = 4$.

Επεξηγήσεις (5/5)

Bombelli presented several more examples of the same type, where in each case he was able somehow to calculate the appropriate values of a and b. He did note, however, that this was not possible in general. If one attempts to solve the system in a and b by a general method, such as substitution, one is quickly led back to another cubic equation. Bombelli also showed

Σχολιασμος: Διάσταση Ιστορίας Και Σχολικού Συστήματος

that complex numbers could be used to solve quadratic equations that previously had been thought to have no solution. For example, he used the standard quadratic formula to show that $x^2 + 20 = 8x$ has the solutions $x = 4 + 2i$ and $x = 4 - 2i$. Although he could not answer all questions about the use of complex numbers, his ability to use them to solve certain problems provided mathematicians with the first hint that there was some sense to dealing with them. Since mathematicians were still not entirely happy with using negative numbers—Cardano called them fictitious and Bombelli did not consider them as roots at all—it is not surprising that it took many years before they were entirely comfortable with using complex numbers.

Η «Άλγεβρα» του Bombelli (Ital.), 1579


L'ALGEBRA
OPERA
Di Rafael Bombelli da Bologna
Diuiſa in tre Libri.
Con la quale ciaſcuno da ſe potrà venire in perfetta cognitione della teorica dell'Arimetica.
Con vna Tauola copioſa delle materie, che in eſſa ſi contengono.
Poſta hora in luce à beneficio delli Studioſi di detta profeſſione.

IN BOLOGNA,
Per Giouanni Roſsi. MDLXXIX.
Con licenza de' Superiori

Εικόνα 1.

Η «Άλγεβρα». Ευχαριστίες στο χορηγό

AL REVERENDISSIMO

MONSIGNOR, IL SIGNOR

ALESSANDRO RVFINI,

VESCOVO DIGNISS.

DI MELFI,

Signore, e Padron ſuo ſempre oſſeruandiſs.

RAFAEL BOMBELLI DA BOLOGNA.

Σύνδεση με προηγούμενα. Ο ρόλος των μιγαδικών αριθμών στα μαθηματικά, περί το 1800

- Εξισώσεις τρίτου και τέταρτου βαθμού
- Πραξεολογια μιγαδικών – Bombelli κλπ
- Μιγαδική εκθετική συνάρτηση
- Εξισώσεις Caychy – Riemann
- Θεμελιώδες θεώρημα της άλγεβρας

Exponential Function

$x \in \mathbb{R}, \quad e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$

$e^{ix} =? \; 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)$

$= \cos x + i\,\text{ημ}x$

ορίζουμε $e^{a+ib} = e^a(\cos b + i \sin b)$

Αποδεικνύεται $e^{z_1+z_2} = e^{z_1} \cdot e^{z_2}$

Εξισώσεις Cauchy – Reimann

$u(x,y)$, $v(x,y)$ πραγματικες συναρτησεις

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Εστω $f(x+iy) = u(x,y) + i\,v(x,y)$

Τοτε υπαρχει η παραγωγος

$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

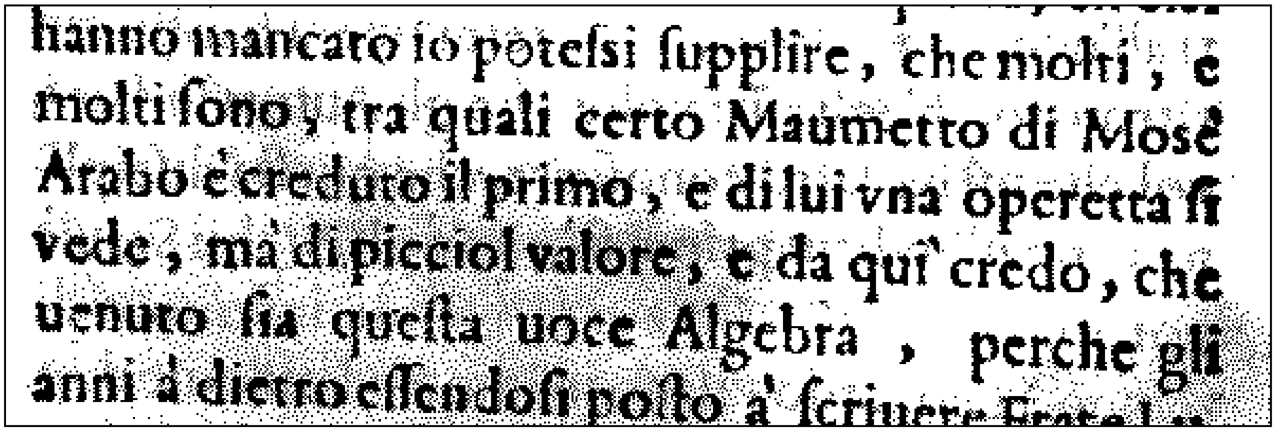
Ο ωραιότερος μαθηματικός τύπος!

- $e^{i\pi} = -1$

Η «Άλγεβρα» του Bombelli (ital.). Λίγα λόγια

- Bombelli's *Algebra* was intended to be in five books. The first three were published in 1572 and at the end of the third book he wrote that [1]:
- *... the geometrical part, Books IV and V, is not yet ready for the publisher, but its publication will follow shortly.*
- Unfortunately Bombelli was never able to complete these last two volumes for he died shortly after the publication of the first three volumes. In 1923, however, Bombelli's manuscript was discovered in a library in Bologna by Bortolotti. As well as a manuscript version of the three published books, there was the unfinished manuscript of the other two books. Bortolotti published the incomplete geometrical part of Bombelli's work in 1929. Some results from Bombelli's incomplete Book IV are also described in [17] where author remarks that Bombelli's methods are related to the geometrical procedures of Omar Khayyam.

Muhàmmad ibn Mussa al-Khwarazmí (Bagdad?, 780 - 850)



hanno mancato io poteſsi ſupplire, che molti, e
molti ſono, tra quali certo Maumetto di Mosè
Arabo è creduto il primo, e di lui vna operetta ſi
vede, mà di picciol valore, e da quì credo, che
uenuto ſia queſta uoce Algebra , perche gli
anni à dietro eſſendoſi poſto à ſcriuere

Η «Άλγεβρα». Αναφορά σε Διόφαντο (1/3)

no creduto, e detto quanti doppo lui hanno ſcrit-
to, ma queſti anni paſſati, eſſendoſi ritrouato
una opera greca di queſta diſciplina nella libraria
di Noſtro Signore in Vaticano, compoſta da un
certo Diofante Aleſſandrino Autor Greco, ilqua-
le fù à tempo di Antonin Pio, & hauendomela fat-
ta vedere Meſſer Antonio Maria Pazzi Reggiano

Η «Άλγεβρα». Αναφορά σε Διόφαντο (2/3)

- http://www-history.mcs.st-andrews.ac.uk/Biographies/Bombelli.html, On one of Bombelli's visits to Rome he made an exciting mathematical discovery. Antonio Maria Pazzi, who taught mathematics at the University of Rome, showed Bombelli a manuscript of Diophantus's *Arithmetica* and, after Bombelli had examined it, the two men decided to make a translation. Bombelli wrote in [2] (see also [3]):- ... [*we*]*, in order to enrich the world with a work so finely made, decided to translate it and we have translated five of the books* (*there being seven in all*)*; the remainder we were not able to finish because of pressure of work on one or other.*

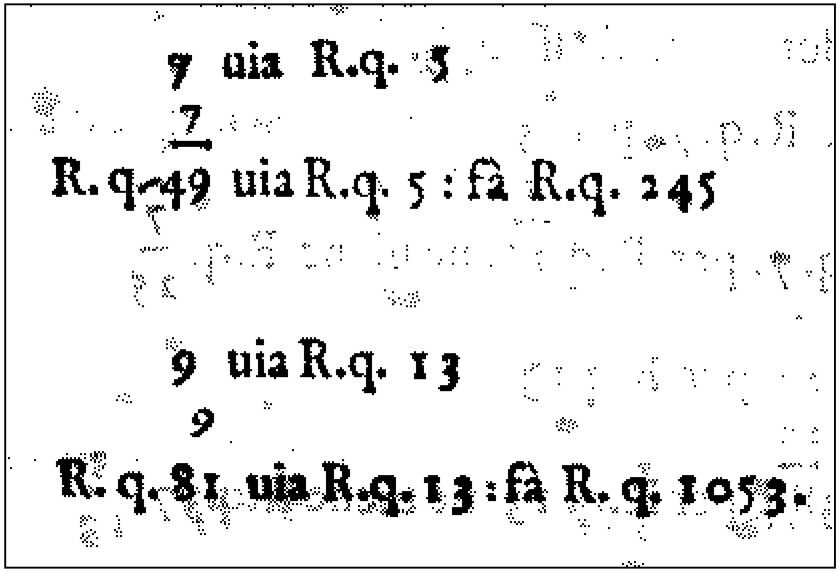
Η «Άλγεβρα». Αναφορά σε Διόφαντο (3/3)

- Despite never completing the task, Bombelli began to revise his algebra text in the light of what he had discovered in Diophantus. In particular, 143 of the 272 problems which Bombelli gives in Book III are taken from Diophantus. Bombelli does not identify which problems are his own and which are due to Diophantus, but he does give full credit to Diophantus acknowledging that he has borrowed many of the problems given in his text from the *Arithmetica.*

Η «Άλγεβρα». Σύμβολα Ριζικών

Radice quadrata	R.q.
Radice cubica	R.c.
Radice quadroquadrata	RR.q.
Radice prima incompoſta, ouer relata	R.p.r.
Radice quadra cubica	R.q.c.
Radice ſeconda incompoſta, ouer ſecon-da relata	R.ſ.r.
Radice quadrata legata con le quantita fra li dui LJ.	R.q. LJ.
Radice cubica legata con le quantità fra li dui LJ.	R.c. LJ.

Η «Άλγεβρα». Πολλαπλασιασμός ριζικών



7 uia R.q. 5

7

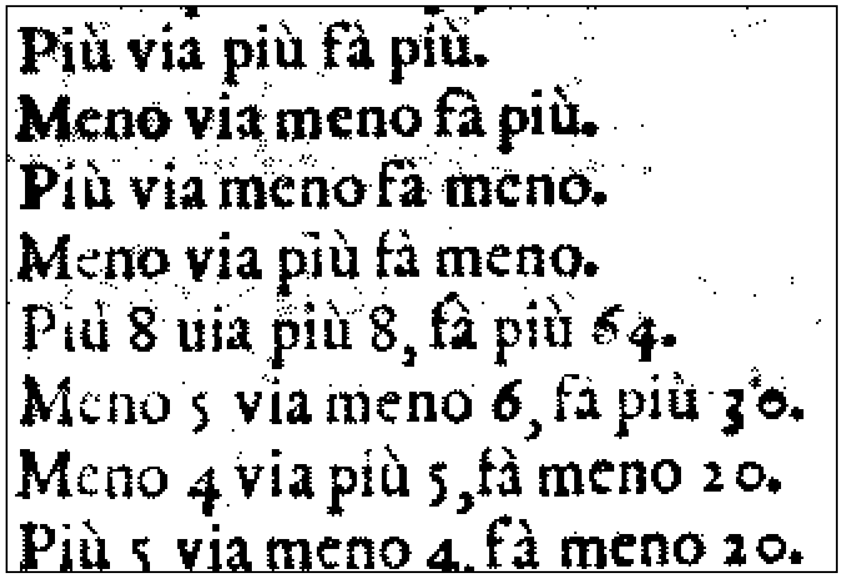
R.q. 49 uia R.q. 5 : fa R.q. 245

9 uia R.q. 13

9

R.q. 81 uia R.q. 13 : fa R.q. 1053.

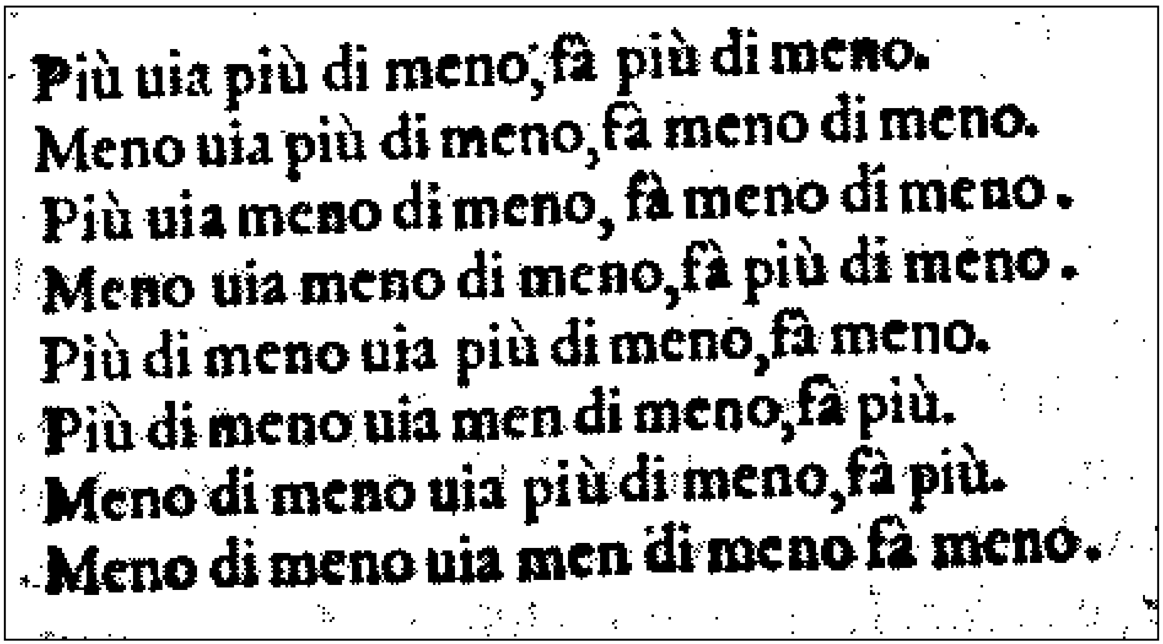
Η «Άλγεβρα». Πολλαπλασιασμός πραγματικών (1/2)


Più via più fà più.
Meno via meno fà più.
Più via meno fà meno.
Meno via più fà meno.
Più 8 uia più 8, fâ più 64.
Meno 5 via meno 6, fà più 30.
Meno 4 via più 5, fà meno 20.
Più 5 via meno 4, fà meno 20.

Η «Άλγεβρα». Πολλαπλασιασμός πραγματικών (2/2)

- Bombelli's *Algebra* gives a thorough account of the algebra then known and includes Bombelli's important contribution to complex numbers. Before looking at his remarkable contribution to complex numbers we should remark that Bombelli first wrote down how to calculate with negative numbers. He wrote (see [2] or [3]):-
- *Plus times plus makes plus, Minus times minus makes plus*
  Plus times minus makes minus, Minus times plus makes minus
  Plus 8 *times plus* 8 *makes plus* 64, *Minus* 5 *times minus* 6 *makes plus* 30
  Minus 4 *times plus* 5 *makes minus* 20, *Plus* 5 *times minus* 4 *makes minus* 20
- As Crossley notes in [3]:- *Bombelli is explicitly working with signed numbers. He has no reservations about doing this, even though in the problems he subsequently treats he neglects possible negative solutions.*
- In Bombelli's *Algebra* there is even a geometric proof that minus time minus makes plus; something which causes many people difficulty even today despite our mathematical sophistication.

Η «Άλγεβρα». Πολλαπλασιασμός μιγαδικών.
i, piu di meno, -i meno di meno


Più uia più di meno, fà più di meno.
Meno uia più di meno, fà meno di meno.
Più uia meno di meno, fà meno di meno.
Meno uia meno di meno, fà più di meno.
Più di meno uia più di meno, fà meno.
Più di meno uia men di meno, fà più.
Meno di meno uia più di meno, fà più.
Meno di meno uia men di meno fà meno.

Η «Άλγεβρα». Πολλαπλασιασμός μιγαδικών

- Bombelli, himself, did not find working with complex numbers easy at first, writing in [2] (see also [3]):- *And although to many this will appear an extravagant thing, because even I held this opinion some time ago, since it appeared to me more sophistic than true, nevertheless I searched hard and found the demonstration, which will be noted below. ... But let the reader apply all his strength of mind, for* [*otherwise*] *even he will find himself deceived.*
- Bombelli was the first person to write down the rules for addition, subtraction and multiplication of complex numbers. After giving this description of multiplication of complex numbers, Bombelli went on to give rules for adding and subtracting them. He then showed that, using his calculus of complex numbers, correct real solutions could be obtained from the Cardan-Tartaglia formula for the solution to a cubic even when the formula gave an expression involving the square roots of negative numbers.

Η «Άλγεβρα». Συμβολισμοί

- Finally we should make some comments on Bombelli's notation. Although authors such as Pacioli had made limited use of notation, others such as Cardan had used no symbols at all. Bombelli, however, used quite **sophisticated** notation.
- It is worth remarking that the printed version of his book uses a slightly different notation from his manuscript, and this is not really surprising for there were problems printing mathematical notation which to some extent limited the type of notation which could be used in print.

Η «Άλγεβρα». Συμβολισμοί

Modern notation	Bombelli printed	Bombelli written
$5x$	$\overset{\underset{\smile}{1}}{5}$	$\overset{\underset{\smile}{1}}{5}$
$5x^2$	$\overset{\underset{\smile}{2}}{5}$	$\overset{\underset{\smile}{2}}{5}$
$\sqrt{4+\sqrt{6}}$	Rq⌊4pRq6⌋	R⌊4pR6⌋
$\sqrt[3]{2+\sqrt{0-121}}$	Rc⌊2pRq⌊0m121⌋⌋	R^3⌊2pR⌊0m121⌋⌋

Η «Άλγεβρα». Επίλογος

- Οι μιγαδικοί εντάχθηκαν στο γενικότερο σύστημα.
- Εξηγήθηκε (ως ένα βαθμό) ο τύπος του Cardano
  συμβολισμός δυνάμεων και ριζικών με αριθμούς.

Bombelli was the last of the Italian algebraists of the Renaissance. His *Algebra*, however, was widely read in other parts of Europe. Two men, one in France and one in the Netherlands, just before the turn of the seventeenth century used both Bombelli's work and some newly rediscovered Greek mathematical works to take algebra into new directions.

Τέλος Υποενότητας

Bombelli και μιγαδικοί αριθμοί

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Αθηνών**» έχει χρηματοδοτήσει μόνο την αναδιαμόρφωση του εκπαιδευτικού υλικού.
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.

Σημείωμα Αναφοράς

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Σημείωμα Αδειοδότησης

Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

Σημείωμα Χρήσης Έργων Τρίτων

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες

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