ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
Εθνικόν και Καποδιστριακόν
Πανεπιστήμιον Αθηνών

Ιστορία νεότερων Μαθηματικών

Ενότητα 3: Η Άλγεβρα της Αναγέννησης

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Περιγραφή Ενότητας

Ιταλοί Αβακιστές. Αλγεβρικός Συμβολισμός. Άλγεβρα στην Γαλλία, Γερμανία, Αγγλία. Εξισώσεις τρίτου και τετάρτου βαθμού. Μιγαδικοί αριθμοί. Εξισώσεις τετάρτου βαθμού και συμμετρίες.

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- Άλγεβρα στη Γαλλία
- Σύμβολα ριζικών
- Άλγεβρα στη Γερμανία
- Rosenfeld Shenitzer, Christoff Rudolff, Michael Stifel, Robert Recorde

Η Άλγεβρα της Αναγέννησης

Η Άλγεβρα στην Γαλλία, Γερμανία, Αγγλία και Πορτογαλία

Εισαγωγή (1/2)

- The medieval economy was also changing in northern Europe during the fourteenth and fifteenth centuries, although developments were generally a bit behind those in Italy. And so mathematics texts began to appear there to meet the new needs of the society. We will consider here the work of **Nicolas Chuquet in France, Christoff Rudolff, Michael Stifel, and Johannes Scheubel in Germany, Robert Recorde in England, and Pedro Nunes in Portugal.** There is much similarity among their works in algebra and also similarities between these works and the Italian algebra of the fifteenth century, so it is clear that these mathematicians all had some knowledge of the contemporaneous work elsewhere in Europe, even though explicit reference to the work of others is generally limited or lacking entirely.
- Απουσία Ισπανίας

Εισαγωγή (2/2)

- But each of them also seems to have some original material. It appears that the knowledge of Islamic algebra had spread widely in Europe by the fifteenth century.
- Each person attempting to write new works used this material and works in algebra from elsewhere in Europe, adapted them to fit the circumstances of his own country, and introduced some of his own new ideas.
- By the late sixteenth century, with the spread of printing, new ideas could circulate more rapidly throughout the continent, and those generally felt to be most important were absorbed into a new European algebra.

Algebra in France, Nicolas Chuquet (c. 1450-c. 1494) (1/2)

- Nicolas Chuquet (d. 1487) was a **French physician** who wrote his mathematical treatise in Lyon near the end of his life. Lyon in the late fifteenth century was a thriving commercial community with a growing need, as in the Italian cities, for practical mathematics. It was probably to meet this need that Chuquet composed his *Triparty (Le Triparty en la Science des Nombres par Maistre Nicolas Chuquet Parisien) in 1484, a work on arithmetic* and algebra in three parts, followed by three related works containing problems in various fields in which the rules established in the *Triparty are used.*
- *These supplementary problems* show many similarities to the problems in Italian abacus works, but the *Triparty itself is on a* somewhat different level in that it is a text in mathematics itself. Most of the mathematics in it was certainly known to the Islamic algebraists and also to Leonardo of Pisa. Nevertheless, since it is the first detailed algebra in fifteenth-century France, we will consider some of its important ideas.

Algebra in France, Nicolas Chuquet (c. 1450-c. 1494) (2/2)

- The first part of the Triparty is concerned with arithmetic. Like the Italian works, it began with a treatment of the Hindu-Arabic place value system and detailed the various algorithms for the basic operations of arithmetic, both with whole numbers and with fractions.
- One of Chuquet's procedures with fractions was a rule "**to find as many numbers intermediate between two neighboring numbers as one desires**." His idea was that to find a fraction between two fractions, one simply adds the numerators and adds the denominators.

Regula Falsi (1/3)

between 1/2 and 1/3 is 2/5, and between 1/2 and 2/5 is 3/7. Chuquet gave no proof that the rule is correct, but he did apply it to deal with finding roots of polynomials. For example, to find the root of $x^2 + x = 39\frac{13}{81}$, Chuquet began by noting that 5 is too small to be a root, while 6 is too large. He then proceeded to find the correct intermediate value by checking, in turn, $5\frac{1}{2}$, $5\frac{2}{3}$, $5\frac{3}{4}$, and $5\frac{4}{5}$ and determining that the root must be between the two last values. Applying his rule to the fractional parts, he next checked $5\frac{7}{9}$, which turns out to be the correct answer.

Regula Falsi (2/3)

- If $a/b < c/d$ then $a/b < (a+c)/(b+d) < c/d$,
- If a, b, c, d, are positive
- If $a/b < c/d$ then $ad < bc$.
- On the other hand: $(a+c)b - a(b+d) = cb - ad > 0$, so
- $a/b < (a+c)/(b+d)$
- Similarly the second inequality follows

Regula Falsi (3/3)

In part two of the *Triparty*, Chuquet applied the rule to the calculation of square roots of numbers that are not perfect squares. Noting that 2 is too small and 3 too large to be the square root of 6, he began the next stage of his approximation procedure by determining that $2\frac{1}{3}$ is too small and $2\frac{1}{2}$ too large. His next several approximations were, in turn, $2\frac{2}{5}$, $2\frac{3}{7}$, $2\frac{4}{9}$, $2\frac{5}{11}$, and $2\frac{9}{20}$. At each stage he calculated the square of the number chosen and, depending on whether it is larger or smaller than 6, determined between which two values to use his rule of intermediates. He noted that "by this manner one may proceed, . . . until one approaches very close to 6, a little more or a little less, and until it is sufficient. And one should know that the more one should continue in this way, the nearer to 6 one would approach. But one would never attain it precisely. And from all this follows the practice, in which the good and sufficient root of 6 is found to be $2\frac{89}{198}$, which root multiplied by itself produces 6 plus 1/39,204."[10] Chuquet evidently was aware of the irrationality of $\sqrt{6}$ and had developed a new recursive

Greek Dichotomy (1/2)

- KATZ. 12.2.1 **France: Nicolas Chuquet, p.391**
- Algorithm to calculate it to whatever accuracy may be desired. He had therefore taken another step on the road to denying the usefulness of the Greek dichotomy between the discrete and the continuous, the final elimination of which was to occur about a century later.
- **Σχόλιο:** Greek Dichotomy?

Greek Dichotomy (2/2)

- The continuous, the **final elimination** of which was to occur about a century later.
- Chuquet also displayed in the second part of his work the standard methods for calculating the square and cube roots of larger integers, one integral place at a time, but as is usual in the discussion of these methods, he did not take the method below the unit.
- He showed no knowledge of the idea of a decimal fraction. If the standard method did not give an exact root, one could choose between calculating using common fractions by his method of intermediates or (and this is the method he preferred) simply not bothering to calculate at all and leaving.

Σύμβολα Ριζικών

the answer in the form $R^2 6$ or $R^3 12$, his notation for our $\sqrt{6}$ and $\sqrt[3]{12}$. Chuquet also used the Italian $\overline{p}$ and $\overline{m}$ for plus and minus, but introduced an underline to indicate grouping. Thus, what we would write as $\sqrt{14 + \sqrt{180}}$, Chuquet wrote as $\underline{R^2 14 \overline{p} R^2 180}$. He proceeded to use this notation with complete understanding through the rest of this second part as he displayed a solid knowledge of computations with radical expressions, both simple and compound, including the necessary rules for dealing with positives and negatives in addition, subtraction, multiplication, and division.

Exponent, actual negative numbers (1/2)

The third part of the *Triparty* was more strictly algebraic, as Chuquet showed how to manipulate with polynomials and how to solve various types of equations. As part of his discussion of polynomials, he introduced an exponential notation for the powers of the unknown, which made calculation somewhat easier than the Italian abbreviations. For example, he wrote 12^2 for what we write as $12x^2$ and, introducing actual negative numbers for the first time in a European work, wrote $\overline{m}12^{2\overline{m}}$ for $-12x^{-2}$. He even noted that the exponent 0 is to be used when one is dealing with numbers themselves. He then showed how to add, subtract, multiply, and divide these expressions (*diversities*) involving exponents (*denominations*) using the standard modern rules, even when one of the exponents is negative. Thus, "whoever would multiply 8^3 by $7^{1\overline{m}}$ it is first necessary to multiply 8 by 7 coming to 56, then he must add the denominations, that is to say $3\overline{p}$ with $1\overline{m}$ coming to 2. Thus, this

Exponent, actual negative numbers (2/2)

multiplication comes to 56^2, and so should others be understood.”[11] Not only did he give this rule, similar to that of one of his Italian contemporaries, but he also justified it. He wrote down in two parallel columns the powers of 2 (beginning with $1 = 2^0$ and ending with $1{,}048{,}576 = 2^{20}$) and the corresponding denomination and then noted that multiplication in the first column corresponded to addition in the second. For example, 128 (which corresponds to 7) multiplied by 512 (which corresponds to 9) gives 65,536 (which corresponds to 16). Because the addition rule of exponents works for numbers, he simply extended it to his diversities. But although he showed that he understood the meaning of negative exponents, his table for numbers did not include them, and, in fact, unlike al-Samaw’al, he made little use of them in what follows.

Ελαφρά επέκτασις

Chuquet also had a few innovations in his equation-solving techniques. First, he generalized al-Khwārizmī's rules to equations of any degree that are of quadratic type, thus going somewhat further than the Italian abacists. For example, he gave the solution of the equation $cx^m = bx^{m+n} + x^{m+2n}$ as

$$x = \sqrt[n]{\sqrt{(b/2)^2 + c} - (b/2)}.$$

Rejection of Negatives (1/2)

Second, he noted that a particular system of two equations in three unknowns has multiple solutions. To solve the system $x + y = 3z$, $x + z = 5y$, he first picked 12 for x and then found $y = 3\frac{3}{7}$ and $z = 5\frac{1}{7}$. Then he picked 8 for y and calculated $x = 28$ and $z = 12$. "Thus," he concluded, "it appears that the number proposed alone determines the varying answer."[12] Finally, although he was not consistent about this, Chuquet was willing under some circumstances to consider negative solutions to equations, again for the first time in Europe. For example, he solved the problem $\frac{5}{12}(20 - \frac{11}{20}x) = 10$ to get $x = -7\frac{3}{11}$. He then checked the result carefully and concluded that the answer is correct. In other problems, however, he rejected negative solutions as "impossible," and he never considered 0 to be a solution.

Rejection of Negatives (2/2)

- The three supplements to the Triparty contained hundreds of problems in which the techniques of that work were applied.
- Many of the problems were commercial, of the same type found in the Italian abacus works, while others were geometrical, both practical and theoretical.
- This work may have been intended as a text, although probably not in a university, but, **unfortunately, the Triparty was never printed and exists today only in manuscript form**.
- Some parts of it were incorporated into a work of Estienne de la Roche (probably one of Chuquet's students) in 1520, but neither this work nor Chuquet's itself had much influence.
- **Σχόλιο: Βαθμιαία Πρόοδος**

Algebra Germany (1/4)

- **Germany: Christoff Rudolff** (sixteenth century)**, Michael Stifel** (1487–1567)**, and Johannes Scheubel**, (1494–1570)
- Germany: Christoff Rudolff, Michael Stifel, and Johannes Scheubel Algebra in Germany first appeared late in the fifteenth century, probably due to the same reasons that led to its development in Italy somewhat earlier. It is likely, in fact, that many of the actual techniques were also imported from Italy. The very name given to algebra in Germany, the Art of the Coss, reveals its Italian origin.
- Coss was simply the German form of the Italian cosa, or thing, the name usually given to the unknown in an algebraic equation. Two of the most important Cossists in the first half of the sixteenth century were Christoff Rudolff (sixteenth century) and Michael Stifel (1487–1567).

Algebra Germany (2/4)

- Rudolff (sixteenth century) and Michael Stifel (1487–1567). Christoff Rudolff wrote his Coss, the first comprehensive German algebra, in Vienna in the early 1520s.
- It was published in Strasbourg in 1525. As usual, the book began with the basics of the place value system for integers, giving the algorithms for calculation as well as a short multiplication table.

Algebra Germany (3/4)

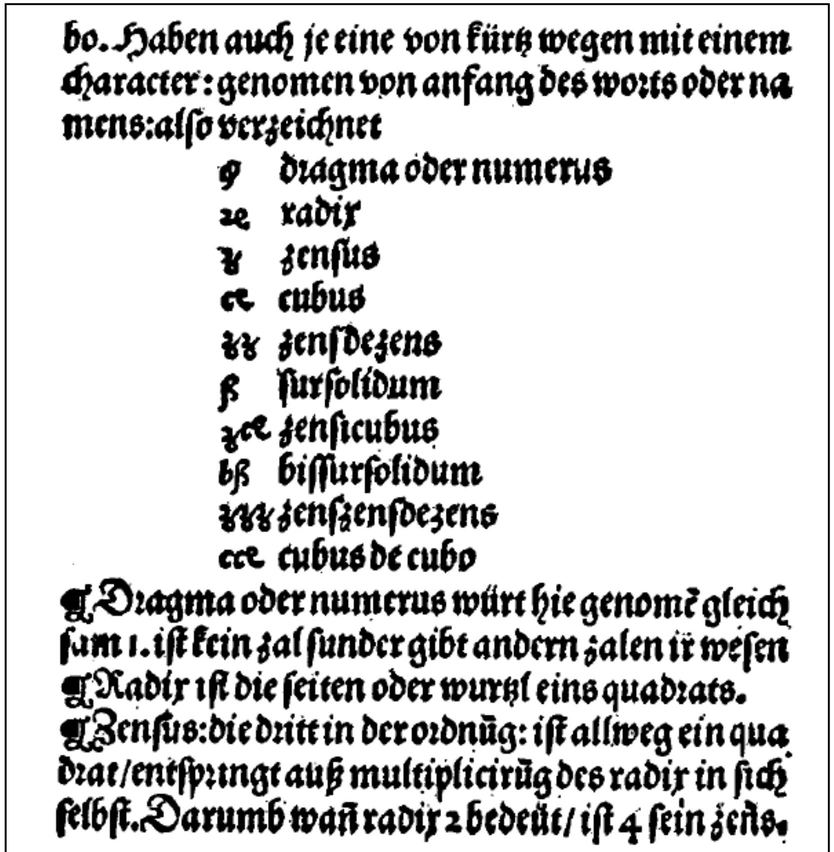
- In a section dealing with progressions, Rudolff included a list of nonnegative powers of 2 alongside their respective exponents, just as Chuquet had done. He also noted that multiplication in the powers corresponded to addition in the exponents. He then extended this idea to powers of the unknown, again as Chuquet had done.
- Although Rudolff did not have the exponential notation of his French predecessor, he did have a system of abbreviations of the names of these powers, where his naming scheme was similar to the Italian multiplicative one (Sidebar 12.1).

Algebra Germany (4/4)

SIDEBAR 12.1 *Rudolff's System for Powers of the Unknown*

dragma	φ	radix	ꝛc $\leftrightarrow x$
zensus	ʒ $\leftrightarrow x^2$	cubus	ꝗ $\leftrightarrow x^3$
zens de zens	ʒʒ $\leftrightarrow x^4$	sursolidum	ß $\leftrightarrow x^5$
zensicubus	ʒꝗ $\leftrightarrow x^6$	bissursolidum	bß $\leftrightarrow x^7$
zenszensdezens	ʒʒʒ $\leftrightarrow x^8$	cubus de cubo	cꝗ $\leftrightarrow x^9$

Cajori History of Mathematical Notations 1 and 2, p. 134 (1/2)


bo. Haben auch je eine von kürtz wegen mit einem
character: genomen von anfang des worts oder na
mens: also verzeichnet

φ	dragma oder numerus
℞	radix
ʓ	zensus
ϲϲ	cubus
ʓʓ	zensdezens
ß	sursolidum
ʓϲϲ	zensicubus
bß	bissursolidum
ʓʓʓ	zenszensdezens
ϲϲϲ	cubus de cubo

¶ Dragma oder numerus würt hie genomē gleich
sam 1. ist kein zal sunder gibt andern zalen ir wesen
¶ Radix ist die seiten oder wurtzl eins quadrats.
¶ Zensus: die dritt in der ordnūg: ist allweg ein qua
drat/entspringt auß multiplicirūg des radix in sich
selbst. Darumb wañ radix 2 bedeüt/ist 4 sein zēns.

Fig. 58.—From Rudolff's *Coss* (1525)

Εικόνα 1.

Cajori History of Mathematical Notations 1 and 2, p. 134 (2/2)

- Zensus, Cesnsus, .
- zens de zens x^2x^2 or $(x^2)^2$?
- zenzicubus $(x^3)^2$, (not x^3x^2)
- zenszensdezens $(((x^2)^2)^2$, not $x^2x^2x^2$
- Cubo de Cubo $(((x^3)^3)$, not x^3x^3
- Surd, Irrational, not logical
- Solidum, solid
- Sursolidum, the first prime number next to three, i.e. 5
- BisSursolidum, the next prime after solidum
- Third Sursolidum, i.e. 11

Sursolidum, Rosenfeld Shenitzer Grant History Of Non Euclidean Geometry.djvu, p. 158 (1/2)

The Latin *surdus*—that gives rise to *surda solida* and *surdisch* (literally: deaf)—is a translation of the Arabic *aṣamm* (dumb, deaf), the Arabic term for the Greek *alogos*—inexpressible. The Greek, Islamic, and Western European scholars used (respectively) the terms *alogos*, *aṣamm*, and *surdus* for irrational roots. Later, *surdus* came to denote irrational numbers. This explains the term *surdic and irrational section*. It means that it is not possible to obtain square and cube roots of x^5 and x^7, that is, such roots of these powers of integers are irrational. Also, we are explicitly told that *sursolidum* is short for *surdum solidum*.

Sursolidum, Rosenfeld Shenitzer Grant History Of Non Euclidean Geometry.djvu, p. 158 (2/2)

To help the reader understand these terms, Rudolff gave as examples the powers of various numbers. He then showed how to add, subtract, multiply, and divide expressions formed from these symbols. Because it is not obvious how to multiply these symbols, unlike the situation in Chuquet's system, Rudolff presented a multiplication table for use with them, which showed, for instance, that ꝛ times ʒ was ce. To simplify matters, he then included numerical values for his symbols. Thus, *radix* was labeled as 1, *zensus* as 2, *cubus* as 3, and so on, and he noted that in multiplying expressions one could simply add the corresponding numbers to find the correct symbol. In this section Rudolff also dealt with binomials, terms connected

“+” και “-”

- By an operation sign, and included, **for the first time in an algebra text, the current symbols of + and – to represent addition and subtraction**.
- These signs had been used earlier in an arithmetic work of 1518 of Heinrich Schreiber (Henricus Grammateus), Rudolff’s teacher at the University of Vienna. Even earlier they had appeared in a work of Johann Widman of 1489.
- There, however, they represented excess and deficiency rather than operations.

Rudolff’s symbol (1/5)

- 1489. There, however, they represented excess and deficiency rather than operations. Rudolff also introduced in his Coss the modern symbol √ for square root.
- He modified this symbol somewhat to indicate cube roots and fourth roots but did not use modern indices. He did, however, give a detailed treatment of operations on surds, showing how to use conjugates in division as well as how to find the square roots of surd expressions such as 27 + √200.
- He also introduced a symbol for “equals,” namely, a period, as in 1 .2 (x = 2). Often, however, he relied on the German gleich.

Rudolff's symbol (2/5)

The second half of Rudolff's *Coss* was devoted to the solving of algebraic equations, but Rudolff used his own eight-fold classification rather than the standard six-fold one. The rule for the solution of each type of equation was given in words and then illustrated with examples. Although Rudolff dealt with equations of higher degree than two in his classes, like Chuquet he included only those that could be solved by reduction to a quadratic equation or by simple roots. Thus, for example, one of his classes was that now written as $ax^n + bx^{n-1} = cx^{n-2}$. The solution given was the standard

$$x = \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{c}{a}} - \frac{b}{2a}.$$

Rudolff's symbol (3/5)

His sample equations illustrating this class included $3x^2 + 4x = 20$ and $4x^7 + 8x^6 = 32x^5$, both of which have the solution $x = 2$. Like the other authors, however, Rudolff did not deal with either negative roots or zero as a root.

Rudolff's symbol (4/5)

After presenting the rules, Rudolff, as is typical, gave several hundred examples of problems that could be solved using the rules. Many are commercial problems dealing with buying and selling, exchange, wills, and money, or recreational problems, including a version of the old 100-birds-for-100-coins problem. Most of the problems, especially the more practical ones, were given as examples of Rudolff's first class of equations, $ax^n = bx^{n-1}$, for which the solution is $x = \frac{b}{a}$. The problems needing a version of the quadratic formula are generally artificial ones, including the ubiquitous "divide 10 into two parts such that . . . ." At the end of the text, Rudolff presented three irreducible cubic equations with their answers

Rudolff's symbol (5/5)

but without giving a method of solution. He simply noted that others who come later will continue the algebraic art and teach how to deal with these. Curiously, on the final page there is a drawing of a cube of side $3 + \sqrt{2}$ divided into eight rectangular prisms. Whether Rudolff intended this diagram to be a hint for the solution of the cubic equation is not known.

Michael Stifel brought out a new edition of Rudolff's text in 1553, nine years after he had published his own, the *Arithmetica integra*.[13] In this latter work, Stifel used the same symbols as Rudolff for the powers of the unknowns, but he was more consistent in using the correspondence between these letters and the integral "exponents." He went further than Rudolff in writing out a table of powers of 2 along with their exponents, which included the negative values -1, -2, and -3 as corresponding to $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, respectively, but he was probably not aware of Chuquet's similar work with negative exponents.

Combining second degree equations (1/4)

Although Stifel, like most of his contemporaries, did not accept negative roots to equations, he was the first to compress the three standard forms of the quadratic equation into the single form $x^2 = bx + c$, where b and c were either both positive or of opposite parity. The solution, expressed in words, was then equivalent to

$$x = \frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 + c},$$

where the negative sign was only possible in the case where b was positive and c negative. In that case, as long as $(\frac{b}{2})^2 + c > 0$, there were two positive solutions. Combining the three cases of the quadratic into one does not seem a major advance, but in the context of the sixteenth century it was significant. It was another step toward the extension of the number concept, although two centuries were to pass before all algebra texts adopted his procedure.

Combining second degree equations (2/4)

- Concept, although two centuries were to pass before all algebra texts adopted his procedure.
- **Stifel's work was also the first European work both to present the Pascal triangle of binomial coefficients and to make use of the table for finding roots (Table 12.1)**. (The triangle itself had been published earlier on the title page of Peter Apianus's Arithmetic of 1527, but Apianus made no use of the triangle in his book.) Stifel noted that he had discovered these coefficients and the root finding procedure only with great difficulty, as he had been unable to find any written accounts of them.
- **Thus, although these coefficients had been used for that purpose in China and in Islamic countries several centuries earlier, the knowledge of this procedure evidently only reached Stifel indirectly**.

Combining second degree equations (3/4)

- Other texts by German authors over the next several decades also made use of the Pascal triangle to find roots. For example, Johannes Scheubel (1494–1570) displayed the triangle in his De numeris et diversis rationibus of 1545 with the standard instructions for calculating its entries.
- Scheubel's book, written in Latin, was evidently aimed at a different audience than the books of Rudolff and Stifel. In particular, he made little effort to include "practical" applications of the material.
- But he did spend many pages working through the method of extracting higher roots using the entries in the Pascal triangle. Although Scheubel's De numeris was not an algebra text, in 1552 Scheubel published such a text, again in Latin.
- This work, Algebrae compendiosa facilisque descriptio was printed, however, in France and was the first algebra work printed there, with the exception of de la Roche's version of Chuquet's Triparty.

Combining second degree equations (4/4)

BIOGRAPHY

Michael Stifel (1487–1567)

Michael Stifel was ordained as a priest in 1511. Reacting to various clerical abuses, he became an early follower of Martin Luther. In the 1520s he became interested in what he called *wortrechnung* (word calculus), the interpretation of words through the numerical values of the letters involved. Through interpreting certain Biblical passages using his numerical methods, he finally came to the belief that the world would end on October 18, 1533. He assembled his congregation in the church on that morning, but to his great dismay, nothing happened. He was subsequently discharged from his parish and for a time placed under house arrest. Because he had now been cured of prophesying, however, he was given another parish in 1535 through the intervention of Luther. Subsequently, he devoted himself to the study of mathematics at the University of Wittenberg and soon became an expert in algebraic methods, publishing his *Deutsche Arithmetica* in 1545, one year after the *Arithmetica integra*. Later in life, however, he resumed his *wortrechnung* and wrote two books on the subject.

Algebra in England Robert Recorde (1510–1558) (1/4)

Robert Recorde (1510–1558)

Robert Recorde graduated from Oxford in 1531 and was licensed in medicine soon thereafter. Although he probably practiced medicine in London in the late 1540s, his only known positions were in the civil service, positions in which he was not notably successful. On the other hand, he did write several successful mathematics textbooks besides *The Whetstone of Witte*, including *The Ground of Arts* (1543) on arithmetic, *The Pathway to Knowledge* (1551) on geometry, and *The Castle of Knowledge* (1556) on astronomy. His works show that he was especially interested in pedagogy. In particular, his books were set in the form of a dialogue between master and pupil, in which each step in a particular technique was carefully explained.

Algebra in England Robert Recorde (1510–1558) (2/4)

12.2.3 England: Robert Recorde

The *Arithmetica integra* and Stifel's 1553 revision of Rudolff's *Coss* were very important in Germany, influencing textbook writers well into the next century and helping to develop in Germany, as had already been done in Italy, mathematical awareness in the middle classes. They also had influence in England, where they were the major source of the first English algebra, *The Whetstone of Witte*, published in 1557 by the first English author of mathematical works in the Renaissance, Robert Recorde (1510–1558) (Fig. 12.2).

Algebra in England Robert Recorde (1510–1558) (3/4)

The Whetstone of Witte had little that was original in technique, because it was based on the German sources and even used the German symbols for powers of the unknown, but there are a few points of interest in the text, which taught algebra to an entire generation of English scientists. First, Recorde created the modern symbol for equality: "To avoid the tedious repetition of these words—is equal to—I will set as I do often in work use, a pair of parallels, or gemow [twin] lines of one length, thus ═══, because no 2 things can be more equal."[14] Second, he modified and extended the German symbolization of powers of

Algebra in England Robert Recorde (1510–1558) (4/4)

the unknown to powers as high as the 80th, setting the integer of the power next to each symbol and noting that multiplication of these symbols corresponded to addition of the corresponding integers. In fact, he showed how to build the symbol for any power out of the square ʒ, the cube ȼ, and various sursolids (prime powers higher than the third) *ß (where * stands for a letter designating the order of the prime). The fifth power is written ß, the seventh power as ^{b}ß (second sursolid), and the eleventh power as ^{c}ß (third sursolid). Then, for instance, the 9th power is written ȼȼ (cube of the cube), the 20th power as ʒʒß (square of the square of the fifth power), and the 21st power as ȼ^{b}ß (cube of the seventh power). Finally, to help students remember the various rules of operation, he gave them in poetic form. His verse giving the procedure for multiplying and dividing expressions of the form ax^n, where the power n is called the "quantity" of the expression, included the standard rule of signs for those operations as well as the rule of exponents:

Algebra in Portugal

- Pedro Nunes (1502–1578)

Τέλος Υποενότητας

Η Άλγεβρα στην Γαλλία, Γερμανία, Αγγλία και Πορτογαλία

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Αθηνών**» έχει χρηματοδοτήσει μόνο την αναδιαμόρφωση του εκπαιδευτικού υλικού.
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.

Σημείωμα Αναφοράς

Σημείωμα Αδειοδότησης

Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

Σημείωμα Χρήσης Έργων Τρίτων

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες

Εικόνα 1: Rudolff's system for powers of the unknown. Algebra in France, Germany, England and Porugal.

Εικόνα 2: Fig. 58, from Rudolff's Coss.