ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
Εθνικόν και Καποδιστριακόν
Πανεπιστήμιον Αθηνών

Ιστορία νεότερων Μαθηματικών

Ενότητα 3: Η Άλγεβρα της Αναγέννησης

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Τμήμα Μαθηματικών

Περιγραφή Ενότητας

Ιταλοί Αβακιστές. Αλγεβρικός Συμβολισμός. Άλγεβρα στην Γαλλία, Γερμανία, Αγγλία. Εξισώσεις τρίτου και τετάρτου βαθμού. Μιγαδικοί αριθμοί. Εξισώσεις τετάρτου βαθμού και συμμετρίες.

Περιεχόμενα Υποενότητας

- Luca Bartolomeo de Pacioli, Girolamo Cardano
- Γενική κατάσταση στην Ευρώπη
- Άβακας

Η Άλγεβρα της Αναγέννησης

Η Άλγεβρα στην Αναγέννηση συνοπτικά

Εισαγωγικά

- ***But of number, cosa [unknown], and* cubo *[cube of the unknown], however they are compounded . . . , nobody until now has formed general rules****, because they are not proportional among them. . . And therefore, until now, for their equations, one cannot give general rules except that, sometimes, by trial, . . . in some particular cases. And therefore when in your equations you find terms with different intervals without proportion, you shall say that the art, until now, has not given the solution to this case, . . . even if the case may be possible.*
- From the *Summa de arithmetica, geometrica, proportioni et proportionalita* of Luca Pacioli, 1494

Luca Bartolomeo de Pacioli (c. 1447–1517)

- **Fra Luca Bartolomeo de Pacioli**:

sometimes *Paccioli* or *Paciolo*; c. 1447–1517) was an Italian mathematician, Franciscan friar, collaborator with Leonardo da Vinci, and seminal contributor to the field now known as accounting. He is referred to as the Father of Accounting and Bookkeeping (he was the first to publish a work on double-entry system of book-keeping).

Summa de arithmetica, geometria, proportioni et proportionalita, 1494 (1/3)

- **Summa de arithmetica, geometria, proportioni et proportionalita** (*Summary of arithmetic, geometry,* **proportions and proportionality**) is a book on mathematics written by Luca Pacioli and first published in 1494. It contains a comprehensive summary of Renaissance mathematics, including practical arithmetic, basic algebra, basic geometry and accounting, written in Italian for use as a textbook.
- The *Summa* is the first printed work on algebra in a **vernacular** language, and it contains the first published description of the **double-entry bookkeeping system**.[1] It set a new standard for writing and argumentation about algebra,[2] and its impact upon the subsequent development and standardization of professional accounting methods was so great that Pacioli is sometimes referred to as the "father of accounting"

Summa de arithmetica, geometria, proportioni et proportionalita, 1494 (2/3)

- The *Summa* and a few others of his books formed the basis of the works of the sixteenth century mathematicians, including Cardano and Tartaglia. Pacioli was something of a careless writer, so much so that Cardano devoted a chapter in his book *Arithmetica* (1539) to dealing with Pacioli's errors.
- Pacioli dedicated 36 short chapters of his *Summa* to bookkeeping, which he entitled *De Computis et Scripturis (Of Reckoning and Writings)*. The Friar did not invent accounting. He merely distilled the practices employed by merchants of Venice at the time. His topics were very much as found in accounting today. Pacioli described keeping journals and ledgers. His ledger included assets (including receivables and inventories), liabilities, capital, income and expense accounts.

Summa de arithmetica, geometria, proportioni et proportionalita, 1494 (3/3)

- He demonstrated year end closing entries and proposed that a trial balance be used to prove a balanced ledger and even had something to say about accounting ethics and cost accounting. As a good bookkeeping instructor should, he warned that no clerk should retire at night before reconciling the debits and the credits, so that they are equal.
- **Quotation of the Day:** [The *Summa's* chapters on bookkeeping were added] "in order that the subjects of the most gracious Duke of Urbino may have complete instructions in the conduct of business, [and to] give the trader without delay information as to his assets and liabilities." – Luca Pacioli

Girolamo Cardano (1501 –1576) (1/2)

- **Gerolamo** (or **Girolamo**, or **Geronimo**) **Cardano** (Italian: [dʒeˈrɔlamo karˈdano]; French: *Jérôme Cardan*; Latin: *Hieronymus Cardanus*; 24 September 1501 – 21 September 1576) was an Italian Renaissance mathematician, physician, astrologer, philosopher and gambler.[1] He wrote more than 200 works on medicine, mathematics, physics, philosophy, religion, and music.[2] His gambling led him to formulate elementary rules in probability, making him one of the founders of the field.
- He was born in Pavia, Lombardy, the illegitimate child of Fazio Cardano, a mathematically gifted lawyer, who was a friend of Leonardo da Vinci. In his autobiography, Cardano claimed that his mother had attempted to abort him. Shortly before his birth, his mother had to move from Milan to Pavia to escape the Plague; her three other children died from the disease.

Girolamo Cardano (1501 –1576) (2/2)

- In 1520, he entered the University of Pavia and later in Padua studied medicine. His eccentric and confrontational style did not earn him many friends and he had a difficult time finding work after his studies had ended. In 1525, Cardano repeatedly applied to the College of Physicians in Milan, but was not admitted owing to his combative reputation and illegitimate birth.
- Eventually, he managed to develop a considerable reputation as a **physician and his services were highly valued at the courts**. He was the first to describe typhoid fever. In 1553 he cured the Scottish Archbishop of St Andrews of a disease that had left him speechless and was thought incurable. The diplomat Thomas Randolph recorded the "merry tales" rumoured about his methods still current in Edinburgh nine years later.[3] Cardano himself wrote that the Archbishop had been short of breath for ten years, and after the cure was effected by his assistant, he was paid 1,400 gold crowns.

Ars Magna, 1545 (1/2)

- The ***Ars Magna*** (Latin: "The Great Art") is an important book on algebra written by Girolamo Cardano. It was first published in 1545 under the title *Artis Magnæ, Sive de Regulis Algebraicis Liber Unus* (*Book number one about The Great Art, or The Rules of Algebra*). There was a second edition in Cardano's lifetime, published in 1570. It is considered[1] one of the three greatest scientific treatises of the early Renaissance, together with Copernicus' *De revolutionibus orbium coelestium* and Vesalius' *De humani corporis fabrica*. The first editions of these three books were published within a two year span (1543–1545).

Ars Magna, 1545 (2/2)

- His account of the discovery of the rule for the algebraic solution of a cubic equation is given in chapter 11 of Girolamo Cardano's *Ars Magna: "Scipio Ferro of Bologna well-nigh* thirty years ago [c. 1515] discovered this rule and handed it on to Antonio Maria Fior of Venice, whose contest with Niccol`o Tartaglia of Brescia gave Niccol`o occasion to discover it.
- He [Tartaglia] gave it to me in response to my entreaties, though withholding the demonstration. Armed with this assistance, I sought out its demonstration in [various] forms. **This was very difficult**.

Γενική κατάσταση στην Ευρώπη (1/6)

- Many changes began to take place in the European economy in the fourteenth century that eventually had an effect on mathematics. The general cultural movement of the next two centuries, known as the Renaissance, also had its impact, particularly in Italy, so it is in that country that we begin our discussion of Renaissance mathematics.
- The Italian merchants of the Middle Ages generally were what today we might call **venture capitalists**. They traveled themselves to distant places in the East, bought goods that were wanted back home, and returned to Italy to sell them in the hope of making a profit. These traveling merchants needed very little mathematics other than the ability to determine their costs and revenues for each voyage.

Γενική κατάσταση στην Ευρώπη (2/6)

- By the early fourteenth century, a commercial revolution spurred originally by the demands of the Crusades had begun to change this system greatly.
- New technologies in shipbuilding and greater safety on the shipping lanes helped to replace the traveling merchants of the Middle Ages with the sedentary merchants of the Renaissance.
- These “new men” were able to remain at home in Italy and hire others to travel to the various ports, make the deals, act as agents, and arrange for shipping.

Γενική κατάσταση στην Ευρώπη (3/6)

- Thus, international trading companies began to develop in the major Italian cities, companies that had a need for more sophisticated mathematics than did their predecessors.
- These new companies had to deal with letters of credit, bills of exchange, promissory notes, and interest calculations. Double-entry bookkeeping began as a way of keeping track of the various transactions.
- Business was no longer composed of single ventures but of a continuous flow of goods consisting of many shipments from many different ports en route simultaneously

Γενική κατάσταση στην Ευρώπη (4/6)

- The medieval economy, based in large part on barter, **was gradually being replaced by a money economy**.
- The Italian merchants needed a new facility in mathematics to be able to deal with the new economic circumstances, but the mathematics they needed was not the mathematics of the quadrivium, the mathematics studied in the universities.
- They needed new tools for calculating and problem solving. To meet this need, a new class of "professional" mathematicians, the ***maestri d' abbaco,****or abacists, appeared in early fourteenth-century Italy.* These professionals wrote the texts from which they taught the necessary mathematics to the sons of the merchants in new schools created for this purpose.

Γενική κατάσταση στην Ευρώπη (5/6)

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Γενική κατάσταση στην Ευρώπη (6/6)

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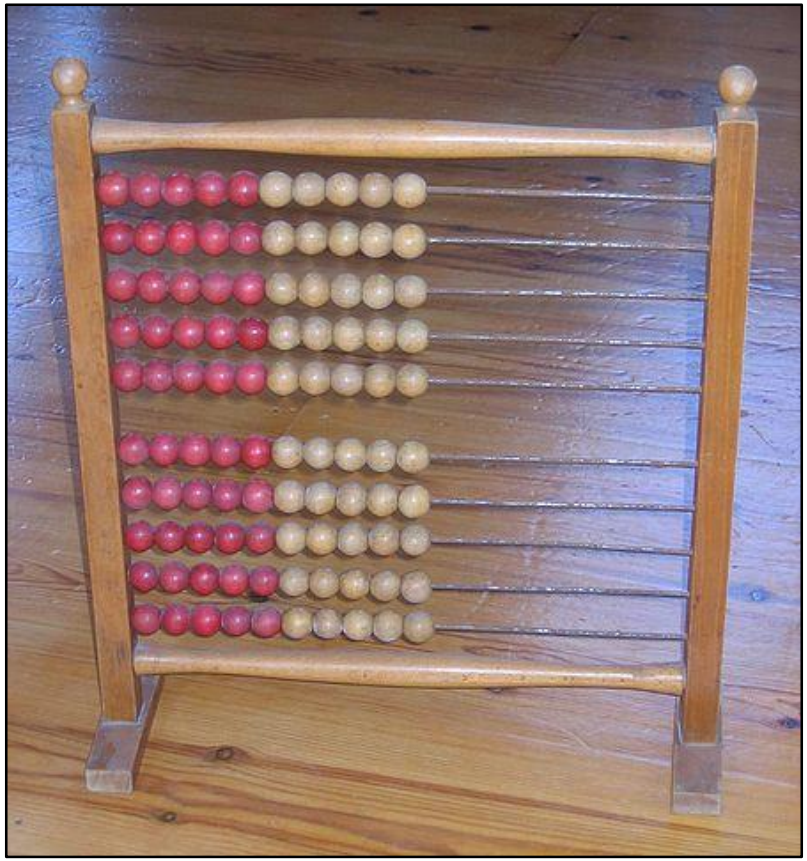
Abacus

- The **abacus** (*plural* abaci or abacuses), also called a **counting frame**, is a calculating tool that was in use centuries before the adoption of the written modern numeral system and is still widely used by merchants, traders and clerks in Asia, Africa, and elsewhere.
- Today, abaci are often constructed as a bamboo frame with beads sliding on wires, but originally they were beans or stones moved in grooves in sand or on tablets of wood, stone, or metal. The user of an abacus is called an *abacist*.

Άβακας

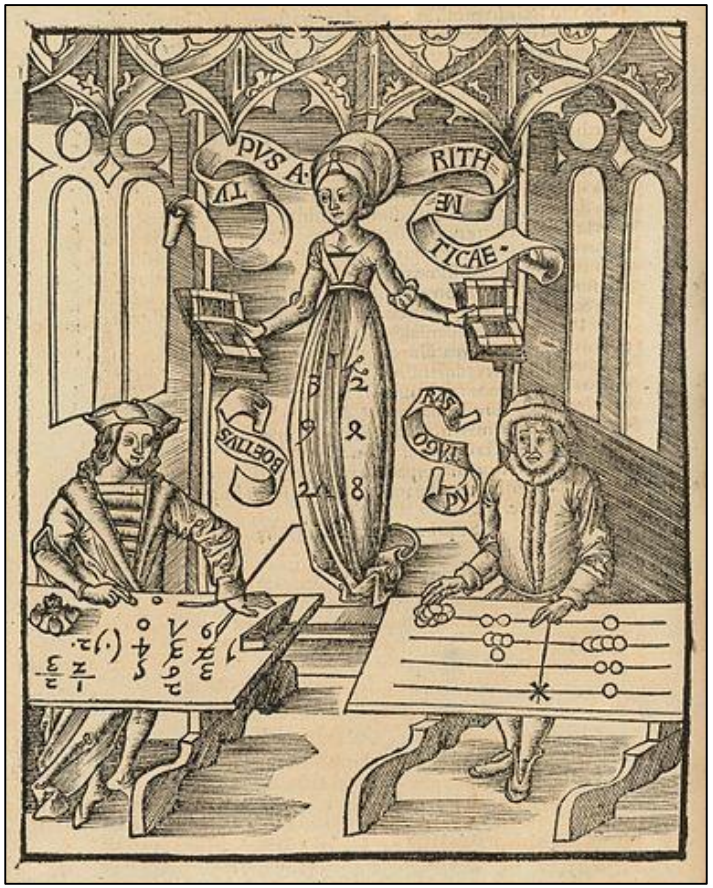
- Ο **άβακας** σημαίνει σήμερα ένα απλό αριθμοόργανο που το χρησιμοποιούμε για την εκτέλεση των βασικών πράξεων (πρόσθεση, αφαίρεση και πολλαπλασιασμό).
- Η λέξη *άβακας* είναι η ελληνική λέξη *άβαξ* που σύμφωνα με τρία αρχαία λεξικά σημαίνει μια πινακίδα, μια σανίδα, κάτι που δεν έχει βάση *Ἄβαξ· κυρίως ὁ μὴ ἔχων βάσιν, καταχρηστικῶς δὲ καὶ ἐπὶ οἱουδήποτε σανιδίου.*
- Δηλαδή αρχικά δεν σημαίνει αριθμητήριο. βλ. <ref> (a) Etymologicum Gudianum, Ed. de Stefani, A. Leipzig: Teubner, 1:1909; 2:1920, Repr. 1965, Alphabetic entry alpha, page 2, line 13; (b) Etymologicum Magnum, Etymologicum magnum, Ed. Gaisford, T. Oxford: Oxford University Press, 1848, Repr. 1967, Kallierges page 2, line 2; (c) Etymologicum magnum genuinum. Symeonis etymologicum una cum magna grammatica. Etymologicum magnum auctum, vol. 1", Ed. Lasserre, F., Livadaras, N., Rome: Ateneo, 1976, Volume 1, page 4, line 31)</ref>.

Early 19th century abacus used in Danish elementary school



Εικόνα 1.

Algorist and an Abacist



Εικόνα 2.

Γενική κατάσταση στην Ευρώπη (4/5)

- So we will discuss the mathematics of the abacists in Italy and, in particular, their algebra.
- Because the commercial revolution soon spread to other parts of Europe as well, the next section deals with late fifteenth- and early sixteenth-century algebra in France, England, Germany, and Portugal.
- But because the major new discoveries in algebra in this time period took place in Italy, **partly in response to Luca Pacioli's statement** as of 1494, cubic equations were in general unsolvable algebraically, we go back to Italy to tell the marvelous story of the ultimate discovery of such a solution in the work of Scipione del Ferro, Niccol`o Tartaglia, Girolamo Cardano, and Rafael Bombelli.

Γενική κατάσταση στην Ευρώπη (5/5)

- All of these algebraists based their work on the Islamic algebras first translated into Latin in the twelfth century.
- But by the middle of the sixteenth century, virtually all of the surviving works of Greek mathematics, newly translated into Latin from the Greek manuscripts that had been stored in Constantinople, were available to European mathematicians.
- The last sections of this chapter are thus devoted to the works of Francois Viete, who used his understanding of Greek mathematics to entirely revamp the study of algebra, and Simon Stevin, who once and for all eliminated the Aristotelian distinction between number and magnitude, in effect giving us our current concept of “number.”

Τέλος Υποενότητας

Η Άλγεβρα στην Αναγέννηση συνοπτικά

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Αθηνών**» έχει χρηματοδοτήσει μόνο την αναδιαμόρφωση του εκπαιδευτικού υλικού.
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.

Σημείωμα Αναφοράς

Σημείωμα Αδειοδότησης

Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

Σημείωμα Χρήσης Έργων Τρίτων

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες

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