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PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHER EDUCATORS: GROWTH THROUGH PRACTICE[1]

ABSTRACT. In this paper we present a study conducted within the framework of an in-service professional development program for junior and senior high school mathematics teachers. The focus of the study is the analysis of processes encountered by the staff members, as members of a community of practice, which contributed to their growth as teacher educators. We offer a three-layer model of growth through practice as a conceptual framework to think about becoming a mathematics teacher educator, and illustrate how our suggested model can be adapted to the complexities and commonalities of the underlying processes of professional development of mathematics teacher educators.

KEY WORDS: community of practice, growth-through-practice, mathematics teacher educators, professional development, sorting tasks

ABBREVIATIONS: MT – Mathematics Teacher; MTE – Mathematics Teacher Educator; MTEE – Mathematics Teacher Educators' Educator

INTRODUCTION

In the past decade there have been several calls for reform in mathematics education that are based on the assumption that well prepared mathematics teacher educators are available to furnish opportunities for teachers to develop in ways that will enable them to enhance the recommended changes. However, there are relatively few formal programs that provide adequate training for potential mathematics teacher educators, let alone research on becoming a mathematics teacher educator. Our study examines the process of becoming a mathematics teacher educator within a professional development program for mathematics teachers. More specifically, we analyze the growth of mathematics teacher educators through their practice.

In our work, the community of mathematics educators is viewed as a community of practice. Theories of communities of practices provide us with tools for analyzing the professional growth and various kinds of learning of teacher educators (Rogoff, 1990; Roth, 1998; Lave & Wenger, 1991). These theories consider teachers' knowledge as developing socially within communities of practice. We take these theories a step further to look at teacher educators' professional knowledge.

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An integral characteristic of the community of practice of mathematics educators is associated with the notion of reflective practice (Dewey, 1933; Schön, 1983; Jaworski, 1994, 1998; Krainer, 2001). Following Dewey's emphasis on the reflective activity of both the teacher and the student, as a means for advancing their thinking, there has been transition from a theoretical perspective of the constructs of reflection and action to a more practical position. Consequently, the notions of reflection on-action and reflection in-action emerged and have been recognized as an effective component contributing to the growth of teachers' knowledge about their practice. In our study, reflection is a key issue for the development of teacher educators.

The work described in this paper was conducted within the framework of a five-year reform-oriented in-service professional development project ("Tomorrow 98" in the Upper Galilee[2]) for junior and senior high school mathematics teachers. The reform of mathematics teaching requires that teachers play an active role in their own professional development (NCTM, 2000). In order to help teachers see new possibilities for their own practice they must be offered opportunities to *(a)* learn challenging mathematics in ways that they are expected to teach and *(b)* engage in alternative models of teaching (e.g., Brown & Borko, 1992, Cooney & Krainer, 1996). Accordingly, the main task of the project staff members was to offer such opportunities for the participating teachers.

Our paper focuses on the professional development of the project's staff members as teacher educators within the framework of the project. We analyze some processes in which the project members engaged as they became more proficient, and the conditions that contributed to their training and professional growth within the community of mathematics educators.

CONCEPTUAL FRAMEWORK

As mentioned earlier, the design of both the program of the project and the research that accompanied it was driven by constructivist views of learning and teaching. According to this perspective, learning is regarded as an ongoing process of an individual or a group trying to make sense and to construct meaning based on their personal experiences and interactions with the environment in which they are engaged. It follows that the community of mathematics educators (i.e., teachers and teacher educators) can be seen as learners who reflect continuously on their work and make sense of their histories, their practices, and other experiences.

In order better to understand the learning processes involved in this community, we adapt and expand two explanatory models of school mathematics teaching – Jaworski's (1992, 1994) teaching triad and Steinbring's (1998) model of teaching and learning mathematics as autonomous systems. We combine these two models into a three-layer model that offers a lens through which to examine the interplay between the learning processes of the different members of the community. More specifically, our model strives at reflecting how "Knowledgeability comes from participating in a community's ongoing practices. Through this participation, newcomers come to share community's conventions, behaviors, viewpoints, and so forth; and sharing comes through participation" (Roth, 1998, p. 12).

EXTENSION OF THE TEACHING TRIAD

Jaworski (1992, 1994) offers a teaching triad, which is consistent with constructivist perspectives of learning and teaching. The triad synthesizes three elements that are involved in the creation of opportunities for students to learn mathematics: the management of learning, sensitivity to students, and the mathematical challenge. Although quite distinct, these elements are often inseparable. According to Jaworski "this triad forms a powerful tool for making sense of the practice of teaching mathematics" (1992, p. 8). We borrow the idea of this teaching triad for describing and analyzing the practice of the leading members of our community, namely, the mathematics teacher educators. Analogous to the way mathematics serves as a challenging content for students, the teaching triad serves as a challenging content for mathematics teacher educators. Accordingly, we consider the teaching triad of a mathematics teacher educator to consist of the challenging content for mathematics teachers (i.e., Jaworski's teaching triad), sensitivity to mathematics teachers and management of mathematics teachers' learning (see Figure 1).

In the current study we distinguish between three different groups of mathematics educators: The mathematics teachers (MTs) who participated in the program, the project staff members who served as mathematics teacher educators (MTEs), and the project director/leading researcher (the first author of this paper), who can be seen as a teacher educators' educator (MTEE). Zaslavsky and Leikin (1999) describe in their table (p. 146, ibid.) some similarities and differences between the teaching triad relevant to each group of the community. It should be noted that within the framework of the project, each group of mathematics educators took different roles alternately, depending on the occasion, either as facilitators of learning or

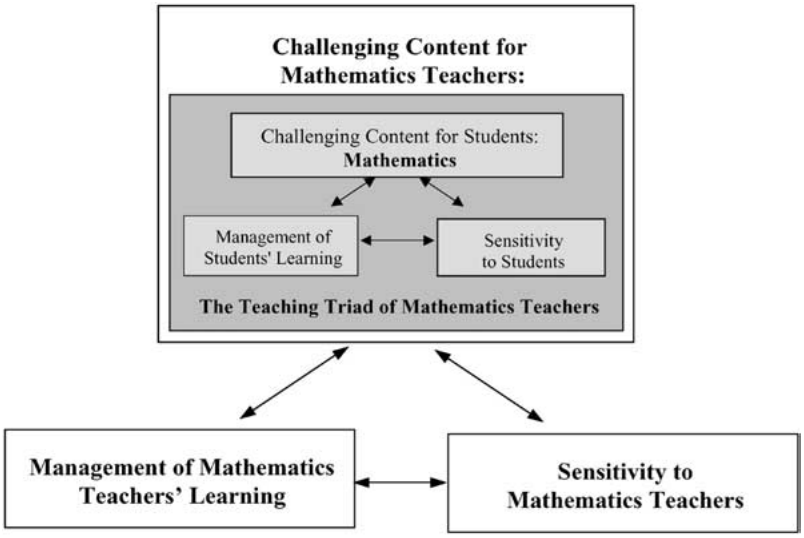

Figure 1. The teaching triad of mathematics teacher educators.

as learners. These dynamic movements between the roles of a facilitator and of a learner fostered many learning-through-teaching processes of the participants, as discussed by Zaslavsky and Leikin (ibid).

A MODEL OF GROWTH THROUGH PRACTICE OF MATHEMATICS TEACHER EDUCATORS

A further description of the learning-through-teaching process may be seen in Steinbring's (1998) model of teaching and learning mathematics. According to this model, the teacher offers a learning environment for his or her students in which the students operate and construct knowledge of school mathematics in a rather autonomous way. This occurs by subjective interpretations of the tasks in which they engage and by ongoing reflection on their work. The teacher, by observing the students' work and reflecting on their learning processes constructs an understanding, which enables him or her to vary the learning environment in ways that are more appropriate for the students. Although both the students' learning processes and the interactive teaching process are autonomous, these two systems are interdependent. This interdependence can explain how teachers learn through their teaching.

Stemming from our reflections on our work within the project, we adapt Steinbring's model and use Jaworski's terminology to help us think about and offer explanations to some ways in which mathematics teachers

(MTs), teacher educators (MTEs) and mathematics teacher educators' educators (MTEEs) may learn from their practice. Parallel to the extension of Jaworski's teaching triad, suggested earlier (Figure 1), we present in Figure 2 a model of teaching and learning as autonomous systems for the different groups of learning facilitators, which is an extension of Steinbring's model (1998). Our three-layer model consists of three interrelated facilitator-learner configurations (dotted, light shaded, and dark shaded), each of which includes two autonomous systems; one system describes the main actions in which the facilitator of learning engages (depicted by circular-like arrows), while the other system describes the main actions in which the learner engages (depicted by rectangular-like arrows).

According to the model depicted in Figure 2, any member of the community may be part of two different configurations at different points of time. For example, a mathematics teacher (MT) may switch roles from a facilitator of students' learning to a member of a group of learners whose learning is facilitated by a mathematics teacher educator (MTE). This is expressed by the two different colors in the rectangle representing MT's knowledge as well as in the two kinds of arrows that come out of this rectangle. The rectangular-like arrow indicates how MTs, as learners, work on their learning tasks (including solving problems that are mathematical, pedagogical or both), make sense of them and construct meaning in subjective ways. By reflecting on their actions and thoughts and by communicating them, MTs develop their knowledge of the teaching triad *through learning*. The other kind of arrow, the circular one surrounding the inner facilitator-learner configuration of our model (i.e., the dotted one), indicates the process underlying MTs development of their knowledge *through teaching*. This inner configuration, which also relates to students' learning, is borrowed from Steinbring (1998). Similarly, MTEs develop their knowledge of the *MTE Teaching Triad* (Figure 1) in two ways: *through learning*, as facilitated by a MTEE, or *through teaching*, when they facilitate MTs learning.

The horizontal bi-directional arrows that come out of the three rectangles representing the knowledge of the different teaching triads indicate two additional kinds of exchange: one exchange has to do with sharing, consulting and exchange of ideas through direct interactions between the different facilitators and learners. The second kind of exchange has to do with a switch from the role of a certain type of facilitator (e.g., MT) to a role of another facilitator (e.g., MTE). The latter exchange may occur as special experiences of a number of MTs on their way to becoming a MTE. In the same way similar switches and exchanges occur between MTEs and MTEEs.

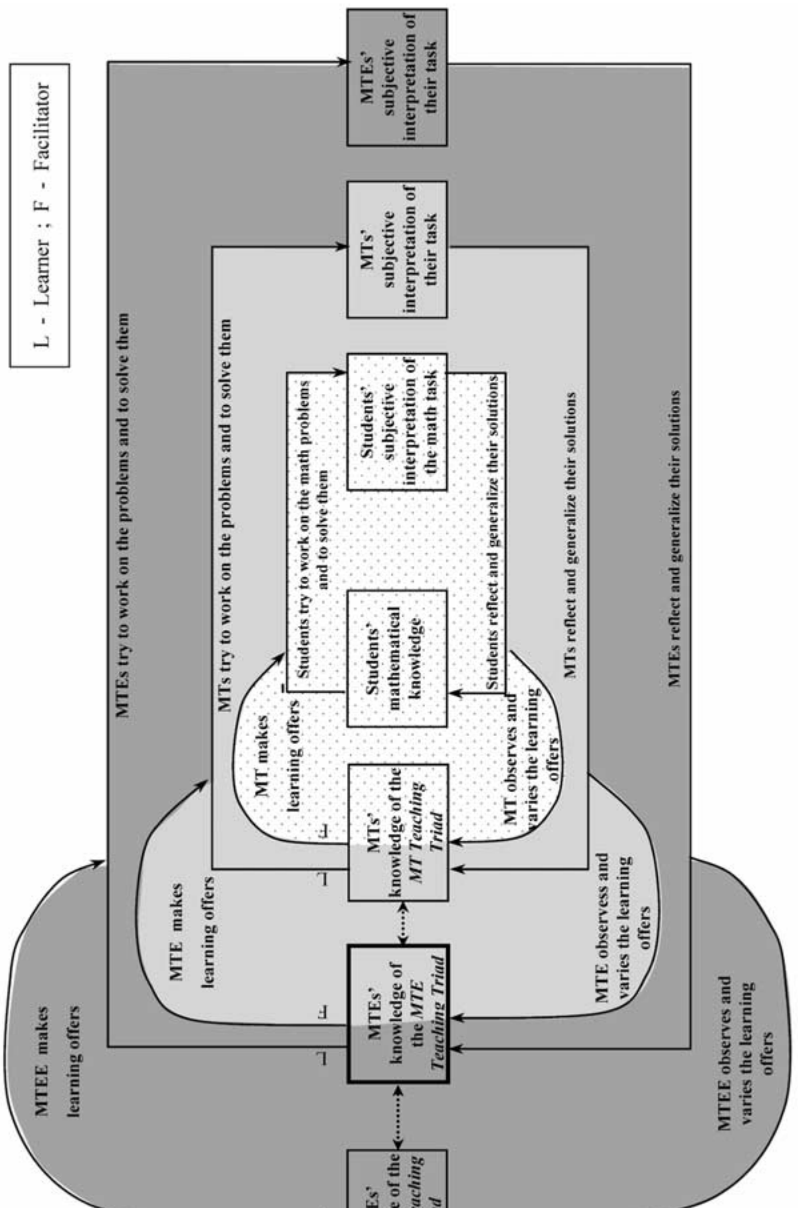

Figure 2. A three-layer model of growth-through-practice of mathematics educators (an extension of Steinbring's (1998) model).

Krainer (1999) points to the unique complexity of the project that is the focus of this paper, particularly in relation to the challenge of interweaving the professional growth of mathematics teachers (MTs) with the growth of mathematics teacher educators (MTEs). The purpose of our paper is to characterize the nature of this complex symbiotic growth and to illustrate how our model applies to practice. This is done using a story through which we gain insight into the underlying processes of the MTEs professional growth. We begin by describing in more detail the overall setting of the project and some characteristics of the team members, followed by the story.

THE GOALS OF THE STUDY

Our three-layer model presented above is actually one outcome of a larger study, in which we attempted to understand and make sense of the ways in which the members of the community developed through their practice. Our goal in this paper was twofold: first, to analyze and understand better the growth through practice of teacher educators as members of the community of mathematics educators. Second, we aimed at testing the descriptive and explanatory power of our theoretical model by applying it to this specific context.

The Overall Setting

The goals and design of the project were very much in line with what Cooney and Krainer (1996) consider as essential components for teacher education programs. More specifically, the project goals included the following:

- Facilitating teachers' knowledge (both mathematical and pedagogical) in ways that support a constructivist perspective to teaching, by:
  - Offering teachers opportunities to experience alternative ways of learning (challenging) mathematics;
  - Preparing teachers for innovative and reform oriented approaches to management of learning mathematics (particularly, the kinds with which they have had very limited experience);
  - Fostering teachers' sensitivity to students and their ability to assess students' mathematical understanding.
- Promoting teachers' ability to reflect on their learning and teaching experiences as well as on their personal and social development.
- Enhancing teachers' and teacher educators' socialization and developing a supportive professional community to which they belong.

Participants

Roth (1998) points to the connection between the length of time for which a community is designed and its goals. Accordingly, our community was designed for a period of five years, with newcomers gradually joining during the first three years.

In total, about 120 teachers participated in the program. The teachers were grouped according to the grade levels they taught (junior/senior high grades) and the year in which they enrolled in the program. Altogether, there were six groups of teachers each consisting of about 20 teachers. During the first three years of the project, two new groups of teachers joined the program each year – one junior high and the other senior high school level.

The teachers who participated in the full program took part for four consecutive years in weekly professional development meetings for six hours per week, throughout each school year. The meetings consisted of a wide range of activities led mainly by the project team. Some of the teachers gradually became more involved in the program and towards their third year assumed responsibility for many of these activities. As the program progressed, the location of the activities shifted from a central regional location into the schools in the region.

The project was designed to enhance the development of the project members hand in hand with the development of the in-service teachers who participated in the program. The design of both the staff enhancement component of the project and the research that focused on the staff members' professional growth stemmed from the project's goals, and was based on two main assumptions: *(a)* Similar to the ways in which teachers learn through their own (teaching) practice (Brown & Borko, 1992; Mason, 1998; Steinbring, 1998; Leikin, Berman & Zaslavsky, 2000), teacher educators learn through their practice; *(b)* There are learning aspects that are fundamentally inherent to the structure and nature of the community of practice, which the project team constitutes (Lave, 1996; Roth, 1998).

The project team consisted mainly of experienced and highly reputable secondary mathematics teachers. Although there were altogether over 20 team members, only 14 were involved in the project from its early stages until its completion. The team members varied with respect to their expertise and experience, one of the characteristics that Roth (1998) considers essential to a community. None of them had any formal training (such as the Manor Program reported by Even, 1999). Some did not have any previous experience in mentoring or teaching other teachers. In the first three years of the project, the tasks of the staff members were mostly directed toward designing and carrying out in-service workshops. Some were in charge of the in-service activities with the teachers, and others facilitated the activities by participating in the weekly in-service meetings or by assisting in the preparation of resources that were required for the in-service meetings.

It should be noted that, at the beginning of the project, there were many staff members who were not very confident of their qualifications as teacher educators, and who expressed a need for guidance by the project director or other more experienced members. Thus, the project director assumed the role of teacher educators' educator (MTEE) in various ways. The staff members expected to gain expertise as teacher educators within the framework of the project, in order to become more competent in their work. It was only towards the end of the project that most of the staff members considered themselves proficient teacher educators.

METHODOLOGY

The methodology employed in the study followed a qualitative research paradigm in which the researcher is part of the community under investigation. It borrows from Glaser and Strauss's (1967) *Grounded Theory*, according to which the researcher's perspective crystallizes as the evidence, documents, and pieces of information accumulate in an inductive process from which a theory emerges. The methods, data collection and analysis grew continuously throughout the progressing study as integral parts of the professional development project. The researcher acts as a reflective practitioner (Schön, 1983) whose ongoing reflectiveness and interpretativeness are essential components (Erickson, 1986). In our case, the researchers were members of the community of practice which they investigated.

Data Collection

Within the grounded theory paradigm, data collection served two interrelated roles. First, it served to investigate the processes involved in the development and growth of mathematics teachers engaged in the project. Second, it served to influence iteratively the design of the project, as the main task of the staff members was to plan and carry out weekly workshops and related activities with the in-service teachers, with no readily available curriculum. Multiple data were collected during the 5 years of the project.

Staff members were asked to provide written and oral reactions to their colleagues. In order to create situations in which less experienced members would learn from more experienced ones in an apprenticeship like manner (Rogoff, 1990) staff members were required, as part of their working load, to observe their colleagues' workshops (including taking part in the workshops' activities when they felt comfortable to do so) and to provide written and oral reactions to their colleagues. These written reactions served three main purposes: to provide feedback to the learning facilitator, to enhance the capacity of the team members to reflect and to trace the development of reflection skills among the team members.

Many of the workshops were videotaped, first in order to extend the possibilities of team members to observe each other at their convenience along the previous line and second, to allow us to analyze the professional development of team members, the changes in their teaching styles, their sensitivity to the teachers, the nature of the challenging mathematical tasks, the structure of the workshops and the management of learning that they facilitated.

In order to foster reflection and self-analysis of the team members they were also required to give written accounts of the workshops for which they were in charge (Borasi, 1999, reports the significance of writing for enhancing reflection). From the research perspective, these written accounts combined with the videotapes helped us follow changes in teachers' reflective abilities. Semi-structured interviews were conducted with the team members (1–2 with each one) fostering reflection on their personal professional growth and the ways in which they relate it to the project's goals and to the various activities in which they were involved. Their reasoning about their involvement in the project provided information about how they perceived their own development within the framework of the project, and helped us identify significant sites in the process of MTE professional development.

Detailed written summaries of staff meetings were available for all staff members. Staff meetings were conducted on a regular and frequent basis. In these meetings, staff members could reflect on their work, share their experiences, consult with their colleagues, and negotiate meaning with respect to the goals and actions of the project. The summaries of the meetings helped us realize commonalities as well as differences in MTEs' positions regarding various issues that were meaningful to the overall progress of the project. They also led us to understand how these positions were modified as a result of interactions between the staff members.

The underlying assumption of the project was that ownership and responsibility, which are indicators of professionalism (Noddings, 1992), would contribute to MTEs' positions in their community of practice. Thus, staff members were continuously encouraged to initiate ideas and suggest new directions and actions within the project. This aspect was also manifested in a series of resource material for mathematics teachers and teacher educators that evolved as a result of identifying and reflecting on mutual interests and successful experiences of the team members. From a research perspective, we were able, using these materials, to identify changes in MTEs' conceptions of the goals and activities of the project, in general, and what constitutes challenging and worthwhile tasks for MTs in particular.

In addition, the project director kept a personal journal including detailed notes of all events and interactions with and among the team members in which she was present. Through these notes, together with other sources, many stories emerged.

To summarize this section, it seems worthwhile to note that the above components that were designed to contribute to MTEs' professional growth address all four dimensions, which Krainer (1998, 1999) considers as describing MTs' professional practice: Action, Reflection, Autonomy, and Networking. In our work, these dimensions refer to MTEs' professional development. From a research perspective, the multiple sources allowed triangulation and supported the validity of our findings.

Data Analysis

As described earlier, the project was documented in numerous ways. The data for this paper were analyzed by the two authors through an inductive and iterative process. Our analysis focused mainly on the MTEs, i.e., the team members, through: their performance (by analyzing videotaped workshops), their utterances that contributed to our understanding of their experiences (by analyzing written self reports of MTEs, protocols of their individual interviews, protocols of the staff meetings) and specific events that seemed meaningful or explanatory to us.

Data analysis focused on two main groups of MTEs who differed with respect to their starting points in the project. One group included members who had some previous experience as MTEs. The other group consisted of very experienced and highly reputable secondary school MTs who did not have any previous experience as MTEs.

The findings, drawn independently, were discussed in order to share the interpretations and meanings that we elicited. In the course of our discussions new ideas came up and new questions were raised. As a result a set of "big ideas" emerged and were formulated – describing the professional development encountered by the above two groups of MTEs in the project. Subsequently, the protocols and videotapes were analyzed again in order to crystallize our conception and understanding of these processes.

Finally we chose to present these big ideas by using two staff members (Tami and Rachel), each representative of a different group of MTEs. Tami had some previous experience as MTE while Rachel had no such previous experience. In addition, their storylines intersected at a particular point in the life of the project, which served as a fruitful context for professional development of MTEs. This kind of intersection was also typical in this context. The participatory nature of the research called for special caution with regard to our personal roles in the project as MTEE and MTE respectively. Our involvements in the episodes chosen for analysis strengthened the reflective elements of our analysis. However, we tried to be aware of the influence of our personal biographies as mathematics educators on our theoretical sensitivity to the research process.

Data Presentation

As mentioned earlier, most of the documentation served both as means for enhancing the professional development of the team members and for the research purposes. Thus, it was used to support the emerging personal and collective stories that portrayed the nature of the processes of growth through practice and participation encountered by the team members. Personal stories have been acknowledged as means of presenting meaningful processes of teaching and learning to teach (Schifter, 1996; Chazan, 2000; Krainer, 2001; Lampert, 2001; Tzur, 2001). Krainer (2001) points to three learning levels that may be connected by stories:

> Firstly, stories provide us with authentic evidence and holistic pictures of exemplary developments in the practice of teacher education. Secondly, through stories we can extend our theoretical knowledge about the complex processes of teacher education. Thirdly, and possibly most important of all, stories are starting points for our own reflection and promote insights into ourselves and our challenges, hopefully with consequences for our actions and beliefs in teacher education (p. 271, ibid.).

One such story that evolved over time had to do with the theme of "cooperative learning and learning to cooperate" (Zaslavsky & Leikin, 1999). In the next section we present another story that is related to the above three learning levels with respect to the growth through practice of MTEs. Indeed, this story, as well as other similar stories that emerged from our study, served as starting points for our reflections and insights, which led us to the development of the theoretical model presented earlier in this paper. By communicating our story here, we try to provide authentic evidence of an exemplary development in the growth through practice of teacher educators.

There are five characters in the story, representing the diversity of the members of the community of mathematics educators in the project: Tami, Alex, Rachel, Hanna, and Keren. Tami and Alex represent experienced MTEs, while Rachel and Hanna represent experienced MTs in transition to becoming MTEs; Keren, represents the MTEE, who often assumed the role of a MTE. As mentioned earlier, we chose Tami and Rachel as the main two characters, through which we convey the interplay and transition from MTE to MTEE (by Tami's storyline) hand in hand with the interplay and transition from MT to MTE (by Rachel's storyline). Thus, our story focuses on the professional development of these two MTEs (Tami and Rachel), while the other characters in the story provide the necessary background and serve to shed light on MTEs' interactions with other members of the community and on their growth through practice. It should be noted that Tami's experience encompassed components of MTs, MTEs, and MTEEs roles, while Rachel's included just MTs and MTEs encounters. In addition, Tami was actively involved in the transitions of Rachel and Hanna from MTs to MTEs. Therefore, Tami's storyline is the dominant part of the story.

THE STORY: *HOW TO SORT IT? OPENING THE TASK AS A TRIGGER FOR OPENING NEW HORIZONS*

The story described in this paper took place during the second year of the project. At that time Tami,[3] who was a team member from the beginning of the project, worked on the preparation of a workshop for senior high school teachers that focused on the role of the domain and range of a function in solving conditional statements (i.e., equations and inequalities). Another staff member, Alex, who joined the team at the end of the first year, worked independently on a workshop with a similar mathematical context. Both of them had similar mathematical background and, based on their personal experiences and professional knowledge, they both considered this topic a problematic one for MTs. At a certain point, Keren (the project director who assumed the role as MTEE) realized that they were both interested in working on the same mathematical topic and suggested that they collaborate and prepare a workshop together. Tami and Alex had different teaching styles; Tami, who had special expertise in developing and implementing cooperative learning approaches in mathematics, suggested managing the workshop in a cooperative learning setting; contrary to Tami's suggestion, Alex was inclined to organize the workshop in a more teacher educator centered fashion, where he would lead the teachers towards the consideration of the use of the domain and range of a function in solving equations and inequalities.

The first stage of collaboration included discussion of and an agreement on the specific mathematical tasks on which they would base the learning offers for the teachers. Thus, they each composed a collection of mathe-

The Task: Please sort the following conditional statements in several different ways. For each of the sorting criteria, please define it and complete the sorting sheet *(Figure 4)*. Mark the order in which the criteria you used for different sorting occured in the course of your work.

1	$\sqrt{x-1}+\sqrt{2-x}=2$	2	$log_2\left(log_{\frac{1}{3}}(x)\right)<0$
3	$\sqrt{x^2+px+1}<0$	4	$2\,sin^2 x-7\,sin\,x+3>0$
5	$\sqrt{x^2-1}-\frac{6}{\sqrt{x^2-1}}=1$	6	$log_2(2^x-1)<x-1$
7	$log_2(log_2(log_2 x))<0$	8	$log(x+1)>log\,x$
9	$\sqrt{x-1}+\sqrt{2-x}=1$	10	$\sqrt{9-x^2}=log_3(\lvert x\rvert-3)$
11	$13\cdot 4^x-2^{x+3}-5=0$	12	$log(x-3)-log(x+9)=log(x-2)$
13	$(tg^2x-1)^{-1}=1+cos\,2x$	14	$\sqrt{2+x-x^2}<x^3-9$
15	$\sqrt{4x-7}<x$	16	$\sqrt{x^2+4x+5}>-1$
17	$\sqrt{x^2-x-12}>x$	18	$\sqrt{x}>x-2$
19	$log(4-x)-log(x-6)=1$	20	$sin\,x+sin\,3x=2$
21	$\sqrt{x-2}+\sqrt{1-x}=2$	22	$log_{cos\,x}\,sin\,x+log_{sin\,x}\,cos\,x=2$

The Setting: It is recommended that you work in small groups of 4 to 6 teachers in a group. Each group has to choose a leader who will be in charge of the progress of the group, and in particular of the completion of the (group) written sorting sheets both on paper and in the form of transparencies that will be presented to the whole group at the end of the small group work. The small groups will be asked to present their completed sorting sheets one at a time. The first group of teachers will present one sorting sheet, which they consider the most interesting. The next group will present one of their sorting sheets that differs from the previously presented one, and so on. After each presentation, the entire group will be asked to react to the proposed criterion, its sub-categories, and the corresponding sets of statements. At the end of the presentations a whole group reflective discussion will be conducted, relating to the task, the cooperative learning setting, and the implication of your learning experiences to your work.

Figure 3. The group assignment – the task and the setting of the workshop.

matical objects – equations and inequalities. The combined set included a total of 40 objects of which they negotiated the selection of a sub-set. Consequently, they ended up with the selection of 22 items (see the final set in Figure 3). The final set of objects varied with respect to two main categories: (1) the kind of family of functions that constituted the equations

Sorting Sheet

Date: ____________________ Group number: ____________________

Members of the group: 1. ____________________ 2. ____________________
3. ____________________ 4. ____________________
5. ____________________ 6. ____________________

Serial no. of the sorting criterion: #______ (according to the order of its occurrence in your group discussion)

Sorting criterion (please define precisely): ____

____

____

Division of the statements into sets according to the sorting criterion:

Description of the common features of the statements in the set	The set of statements*
(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	

*Notes: 1) A statement may belong to more than one set;
2) A statement may belong to none of the sets.

Figure 4. A sorting sheet for the group assignment (for details see Figure 3).

and inequalities and (2) the kind of role the domain and range of functions played in the solution processes.

At the second stage, Tami and Alex discussed the management of the teachers' learning. Tami, who felt very strongly that the use of a cooperative learning setting would be significant, convinced Alex, who had little prior experience in using cooperative learning methods, to go along with her. Thus, Tami and Alex began to design a structured cooperative learning workshop according to the *exchange of knowledge* method that Tami had previously developed (Leikin & Zaslavsky, 1999). For this purpose she proposed to group the 22 equations and inequalities according to the families of functions (category 1 above), while Alex insisted on grouping them according to the role of the domain and range of the functions in solving the conditional statements (category 2 above). Tami argued that an initial grouping of the 22 conditional statements according to the families of functions would allow the teachers to infer the role of the domain (or range) when solving the equations and inequalities. On the other hand, Alex wanted to make sure that the different roles of the domain and range were made explicit. At this point, Tami and Alex decided to present the two alternative suggestions to Keren.

This meeting yielded another, more open, approach based on Keren's expertise and professional experience: instead of grouping the equations and inequalities in advance, the decision was made to ask the teachers to sort the 22 equations and inequalities openly, in as many ways with which they could come up (e.g., Silver, 1979; Cooney, 1994). This change in the mathematical task implied a change in the management of learning from a structured method to a more open setting in which the cooperation was fostered by the nature of the challenging mathematical task rather than by structured role assignments (see Figures 3 & 4). After some adaptation of Keren's suggestion to their personal styles, Tami and Alex felt able to carry out the revised plan.

They conducted two consecutive workshops, the first one with 19 senior high school MTs who were in their second year of enrollment in the project, and the second with 22 senior high school MTs who were in their first year in the project. In both workshops a number of staff members were present (among them were Rachel and Hanna, who had just joined the MTE team). Rachel and Hanna participated in the first workshop and then observed some of the group activities in the second workshop. Altogether there were 9 small groups who worked according to the group assignments described in Figure 3: in the first workshop the work was organized in 4 small groups; in the second one there were 5 small groups.

Generally, the teachers in both workshops began sorting by what we term *surface features* (i.e., features that can be observed without solving the statements), and only after a while turned to more *structural features* (that may be identified only by solving the statements). Tami and Alex had anticipated that the teachers would suggest only two ways of sorting the given conditional statements: according to the families of functions that appear in the statements (a surface feature) and according to the roles of the domain and range of the functions in solving the equations and inequalities (a structural feature). Much to their surprise, overall the 9 groups of teachers proposed 11 different criteria for sorting the statements, as shown in Figures 5 & 6. Note that the shaded areas in Figure 5 (criteria *ii*, *ix* & *x*) indicate sorting according to the criteria initially planned by Tami and

	The Sorting Criterion	The Common Features of the Statements in a Set Generated According to the Sorting Criterion	The Set of Statements (the numbers correspond to Figure 3)
Surface Features (observed without solving the statements)			
i	The type of the conditional statement	Equations	1, 5, 9, 10, 11, 12 13, 19, 20, 21, 22
		Inequations	2, 3, 4, 6, 7, 8, 9, 14, 15, 16, 17, 18
ii	The type of function that is involved in the statement	Quadratic or Cubic (non-linear polynomial)	3, 4, 5, 10, 14, 16, 17,
		Absolute-Value	10
		Square-Root	1, 3, 5, 9, 10, 14, 15, 16, 17, 18, 21
		Trigonometric	4, 13, 20, 22
		Exponential	6, 11
		Logarithmic	2, 6, 7, 8, 10, 12, 19, 22
		Combined	4, 5, 6, 10, 14, 16, 17, 22
iii	The right side of the statement	A number (a constant function)	1, 2, 3, 5, 7, 9, 11, 16, 19, 20, 21, 22
		An algebraic expression (a non-constant function)	6, 8, 10, 12, 13, 14, 15, 17, 18
iv	The level of the task (according to the school curriculum)	3 units	12, 14, 18
		4 units	2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16, 18, 19, 20
		5 units	All the statements
v	The presence of parameters	With parameters	3
		Without parameters	All other (not 3)
Structural Features (identified by solving the statements)			
vi	The domain of the statement	R – all real numbers – the number line	4, 11, 16, 20
		∅ - the empty set	10, 19, 21
		An infinite set of real numbers: • a ray or an interval	1, 2, 6, 7, 8, 9, 12, 14, 15, 18
		• a disjointed set	4, 5, 13, 17, 22
		Depends on the values of a parameter	3
vii	The most convenient representation for the solution	Graphic representation	12, 14, 15, 17, 18
		Algebraic representation	All others
viii	The solution technique (for algebraic solutions)	Squaring the sides of the equation (or inequation)	1, 5, 9, 15, 17, 18, 22
		Substituting a new variable	4, 5, 11
		Using the monotony property of the functions involved	2, 7, 8
ix	Approaches leading to an immediate solution	Using the domain	10, 19, 21
		Using the domain and the monotony property	8
		Using the range	3, 20
		Combining the range and the domain	16
		Checking the values at the endpoints of the domain	1, 9
x	The influence of the domain or range on the solution set	The domain is empty set hence the solution set is empty	10, 19, 21
		The domain restricts the solution set	2, 6, 7, 8, 13, 14, 15, 17, 22
		No domain restriction on the solution	4, 11, 16, 20
		The range restricts the solution	1, 3, 4, 5, 11
xi	The number of solutions	No solution	1, 3, 10, 12, 14, 19, 20, 21
		A finite number of solutions	5, 9, 11
		An infinite number of solutions: • A ray	8, 15, 17, 18
		• A segment	2, 6, 7
		• A line	16
		• A discrete set	4, 13, 22

Figure 5. Teachers' shared classification of the statements – a summary of the first two workshops.

Alex, whereas, criterion *ix* was originally included by Tami and Alex as a sub-category of criterion *x*.

The number of different sorting criteria employed by the small groups varied as follows: three criteria (by 5 groups); four criteria (by 2 groups); six criteria (by one group); and seven criteria (by one group). There was only one sorting criterion which all 9 groups raised (criterion *ii*, which

The Sorting Criterion		No. of Small Groups that Suggested the Criterion		
		1st Workshop (4 Small Groups)	2nd Workshop (5 Small Groups)	Total No. of Small Groups (9)
Surface Features (observed without solving the statements)				
i	The type of the conditional statement	3	3	6
ii	The type of function that is involved in the statement	4	5	9
iii	The right side of the statement	0	1	1
iv	The level of the task (according to the school curriculum)	1	0	1
v	The presence of parameters	1	0	1
Structural Features (identified by solving the statements)				
vi	The domain of the statement	0	2	2
vii	The most convenient representation for the solution	3	1	4
viii	The solution technique (for algebraic solutions)	0	3	3
ix	Approaches leading to an immediate solution	2	1	3
x	The influence of the domain or range on the solution set	1	0	1
xi	The number of solutions	1	3	4

Figure 6. Distribution of teachers' classification of the statements.

was one of the two originally planned). Another rather popular (surface) criterion was suggested by 6 groups (criterion *i*). The rest of the criteria were raised by at most 4 groups of MTs. Interestingly, the main category that Tami and Alex saw as the target concept of the workshop (criteria *ix* & *x*) was addressed only by 3 groups, none of which fully addressed all the sub-categories connected to the role of the domain and range in solving conditional statements. In addition, for the criteria that were suggested by more than one group, there were differences in many cases in the sub-categories specified by the MTs.

At the end of the first workshop, Keren took part as facilitator of the reflective whole group discussion, while Tami followed this role in a similar manner in the second workshop. These concluding discussions focused on the mathematical challenge of the task, the management of this cooperative learning setting, and the implication of the MTs' learning experiences to their knowledge of learners – themselves as well as their students. With respect to the mathematical challenge, the teachers claimed that at first the task appeared to be rather simple and straightforward. However, they realized as they continued working on the task that it offered multiple levels of difficulty and complexity with which they had to deal. There was a consensus that although they were not implicitly required to solve all the conditional statements, they were driven to do so as they proceeded in the task, moving from surface features to structural ones. Even those who had a particularly sound mathematical background found the task challenging. There was an agreement that the small groups' shared ideas and findings led each group to the recognition of additional criteria and sub-categories. Special attention was given to the specifics of the set of statements, which were regarded by the MTs as rich, profound, and instrumental in enlightening the role of the domain and range in solving various conditional statements. Several MTs expressed disappointment in the fact that no small group, on their own, reached the complete sorting scheme that was generated by the entire group (as shown in Figures 5 & 6).

With respect to the cooperative learning setting that was managed in the workshop, the groups reported that they encountered genuine cooperative work that was fostered by the nature of the assignment (see Figure 3). One of the most common ways in which they collaborated was by dividing the set of statements to be solved among the members of the small group. This, in their opinion, increased the efficiency of the group's work, and allowed them to focus on wider and deeper aspects of the statements compared with those they could have reached individually. The need to present their results to the whole group was an incentive to their progress. There were a number of MTs who admitted that, at first, they had resented the requirement to work cooperatively in small groups, since they always preferred to work individually and did not believe in cooperative learning. However, as the workshop progressed, they became involved in the group interactions and felt that they had contributed to the group and that, at the same time, they had gained insights through these interactions. With respect to the knowledge of learners the teachers pointed to what they considered an effective learner-centered environment, which was enhanced by the task and setting. For them it was a meaningful manifestation that such an environment can be implemented and be influential to them as learners. Some of them expressed their willingness to try out a similar activity with their students. Others argued that it would take them too long to design such a learning environment for their students, although, in reflecting on their own learning claimed that they would have liked to provide their students with similar experiences.

At the end of the workshops, when Tami and Alex reflected on the events it turned out that they both felt they had learned a lot. They considered the most critical in the entire process of designing and carrying out the workshop the exchange of ideas with Keren and her suggestion to modify their original task to an open-ended sorting task. What seemed to them at first as a rather minor change, proved to make a cardinal difference. They realized the potential of this kind of task in enhancing the professional growth of MTs with respect to the MTs' teaching triad. They attributed this to the cognitive nature of the task as well as to the openness of the setting. Tami claimed that she had gained appreciation of the power of open-ended mathematical tasks in enhancing MTs' ability to construct shared mathematical and pedagogical knowledge, with very little interference of the MTEs. In addition, Tami explicitly mentioned how her observation of the way Keren facilitated the concluding whole group discussion at the first workshop (as well as observing Keren in other similar situations) had influenced the way she conducted the discussion at the end of the second workshop. Alex expressed a similar view of the above events and their impact on his attitude towards his role of MTE.

As mentioned earlier, Rachel and Hanna were two staff members (MTEs) who took part in these two workshops, first as learners and then as observers. Their participation in the second workshop was initiated by Rachel, who offered to take part in it as an observer and to provide Tami with feedback about her management of MTs' learning. Rachel's reflective discussions with Tami addressed several perspectives which included: the mathematics in which the MTs engaged, the impact of the openness of the task on the mathematical discussions that took place, the interactions and cooperation between the MTs, Tami's flexibility throughout the workshop, and Tami's way of conducting the whole-group discussion.

Together with the MTs, Rachel and Hanna developed an appreciation of the potential of sorting tasks for enhancing mathematical understanding as well as for creating an open learning environment. Consequently, Rachel and Hanna joined Tami and became involved in further endeavors to enhance teachers' implementation of sorting tasks in their classrooms. This collaboration began by extensive guidance that Tami provided for Rachel and Hanna in preparation for conducting a similar workshop on their own. Tami observed this workshop and met with them after it was over to reflect together on the MTs engagement in the sorting tasks and on Rachel's and Hanna's collaborative roles as MTEs. Before conducting more such workshops, Rachel turned to Tami for further advice and suggestions on how to lead an improved workshop.

These iterative experiences led to the design of a series of workshops in which the MTs collaboratively developed sorting tasks in various school mathematics topics for their own students. Following these workshops, there were MTs who began applying these tasks in their classes, and shared their implementation experiences with their colleagues and with staff members at the weekly meetings. A year later, Rachel offered, on her own initiative, to conduct her adaptation of the series of workshops on sorting tasks with a new group of MTs.

ANALYSIS AND DISCUSSION: THE MODEL AND THE STORY

The purpose of this section is to demonstrate the explanatory power of our three-layer model of growth through practice. For this purpose, we

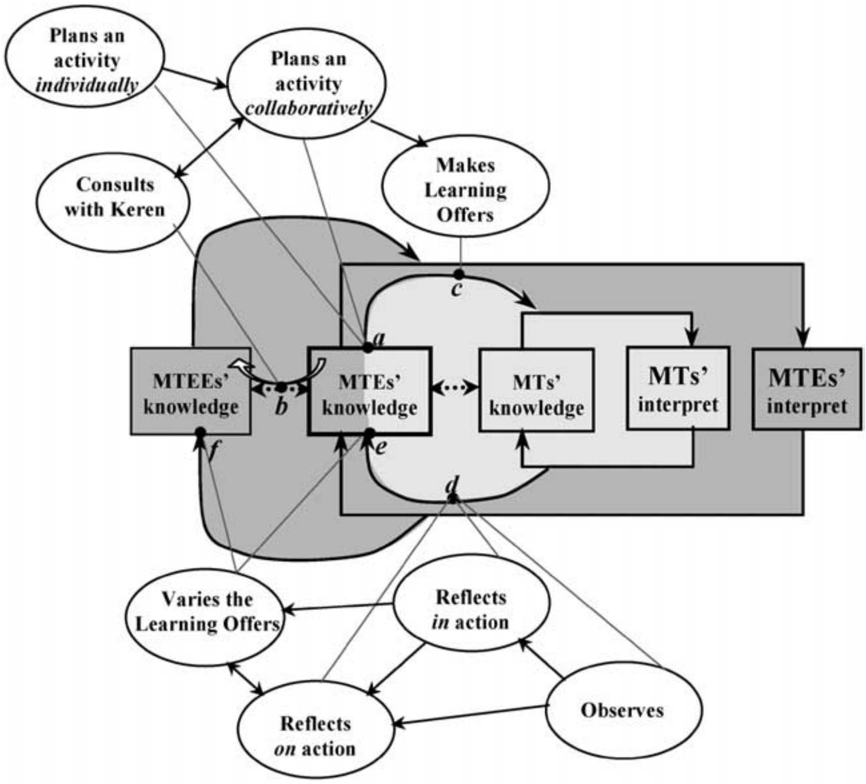

Figure 7. Adaptation of the model of MTEs' growth-through-practice to Tami's storyline.

offer an analysis of our story in light of the model. This analysis conveys the complexities and commonalities of the underlying processes of professional development of MTEs within the framework of the project. The participants of the story are seen as members of a diverse *community of practice* in which its various participants shift roles and continuously engage in learning and facilitating activities.

We turn to an analysis of the professional growth through practice of Tami and Rachel – the key characters of our story – who represent two typical storylines of MTEs' development. Our analysis indicates the connections between their professional development and the interactive nature and structure of the community of practice to which they belonged. We attribute these connections to the different interactions between the various kinds of members of the community of practice – MTs, MTEs, and MTEE – at the three layers of our model. Figures 7 and 8 depict the main benchmarks of Tami's and Rachel's storylines respectively along our model.

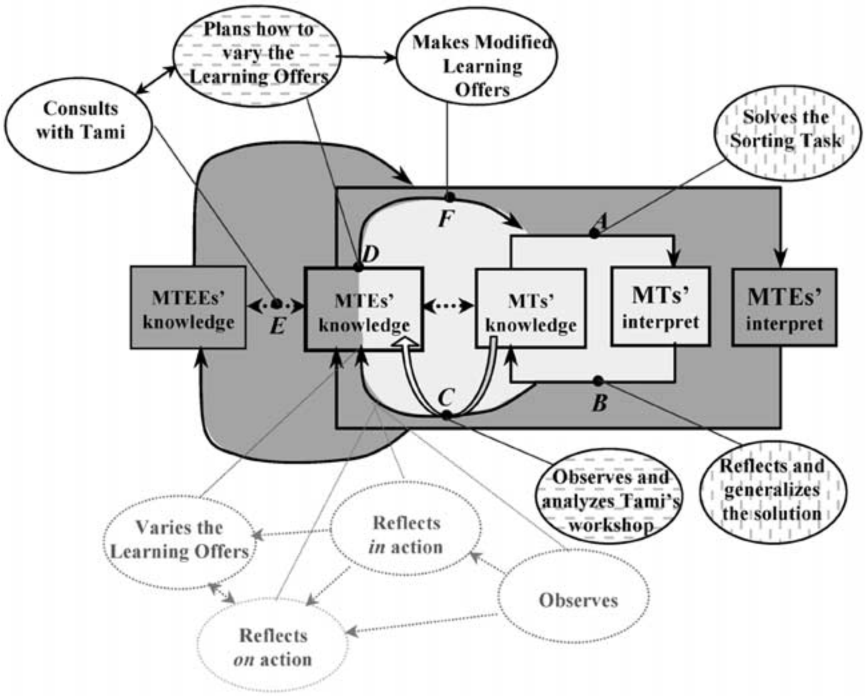

Figure 8. Adaptation of the model of MTEs' growth-through-practice to Rachel's storyline.

The first part of the story exhibits a representative type of interaction between two *collaborating* MTEs with a *similar expertise*, i.e., ***MTE-MTE interactions***. Tami's first benchmark (see ***a*** in Figure 7) involves her collaborative interactions with Alex – another MTE with a similar background – in the course of the planning stage. Tami's initial design of the target workshop originated from Tami's MTE's knowledge of MTE's teaching triad that consisted of both a strong mathematical background and her expertise in cooperative learning methods. The collaborative process of preparing this workshop on the selected mathematical topic – the role of domain and range of function in solving equations and inequalities – with another MTE (Alex) presented Tami with a doubtful situation. This case was similar to many other situations in which MTEs, who worked on a workshop collaboratively, realized that there were differences between their own and their colleagues' positions. We found these differences to be complementary, competing, or contradictory. These doubtful situations drove MTEs to try to reach, by discussion of their different positions, a shared position for preparing the workshop of common interest. This, in return, facilitated their learning.

The next part of the story represents MTEs' learning through *consulting with MTEE*, i.e., *through* ***MTE-MTEE interactions***. Tami's decision to consult with Keren at the initial planning stage (see ***b*** in Figure 7) was motivated by her disagreement with Alex. Such doubtful situations led other staff members in various situations in the project to consult with more experienced members. The exchange of ideas between Tami and Alex and their brain-storming with Keren enhanced Tami's knowledge of the MTE's teaching triad iteratively by adding sorting tasks to her repertoire of potential learning tasks for teachers. Consequently this led to an improved plan of the learning offers that Tami facilitated for the MTs.

The third part of the story, that includes the two workshops that Tami conducted, represents MTEs' learning through *implementation* of the workshop with MTs, i.e., through ***MTE-MT interactions***. This part involved Tami's learning from teachers' ideas through her interactions with them in the course of providing the intended learning offers for MTs and observing their work (see ***c*** and ***d*** in Figure 7). During the first workshop Tami was surprised to realize the numerous approaches of the MTs to the sorting task. By observing the teachers' work on the problems and reflecting in and on action (see ***d*** in Figure 7), Tami became more sensitive to MTs' ways of thinking and became more aware of what may be expected of MTs. She also became convinced of the potential of sorting tasks as a vehicle for professional development. This learning of Tami led her to vary the learning offers in the second workshop (see ***e*** in Figure 7). Note that Tami's learning through observing included her observation of the whole group discussion that was led by Keren (the MTEE) in the first workshop. This part can be seen as involving ***MTEE-MTE-MT interactions***.

The fourth part of the story that deals with Tami's collaborative work with Rachel exhibits another type of ***MTE-MTE interactions***, i.e., interactions between two *collaborating* MTEs with *differing expertise*. Interactions of this type occurred when a more experienced MTE *mentored* a less experienced MTE. In many cases an experienced MTE, after designing and implementing a workshop and encountering the learning entailed in this process, began preparing other MTEs to conduct this workshop in future occasions. This can be considered as a transition stage in which an experienced MTE began taking the role of a MTEE. We analyze the learning that occurred in the course of such interactions from two perspectives: The learning of the more experienced MTE (in this case – Tami), and the learning of the less experienced MTE (in this case – Rachel).

After gaining expertise in planning and running the above workshops, Tami was prepared to mentor other less experienced MTEs (e.g., Rachel and Hanna). In the beginning, she provided extensive guidance to Rachel and Hanna in conducting the workshop. After they had gained their first experience in its implementation, Tami offered them some additional advice towards their second round of implementation. By this process Tami developed her MTEE's knowledge of MTEs learning (see ***f*** in Figure 7 and ***E*** in Figure 8). Overall, the model illustrates how Tami's MTE's knowledge developed and transformed into a MTEE knowledge (see the special arrow above ***b*** in Figure 7).

Rachel's development as a MTE started by participating in Tami's first workshop ***as a MT*** (see ***A*** in Figure 8), and reflecting on her own as well as on her colleagues' work during that workshop (see ***B*** in Figure 8). This interaction with Tami led first to the growth in Rachel's MT's knowledge. Following this experience, Rachel offered to participate in the second workshop as an *observer* and to provide Tami with feedback about the management of the workshop and the learning processes the MTs experienced. Through observation of and reflection on the second workshop (see ***C*** in Figure 8), her knowledge evolved further. In her subsequent interactions with Tami surrounding the second workshop, they both reflected on and analyzed the workshop from several perspectives (see earlier, in the story). Through the exchange with Tami, Rachel developed her MTE's knowledge and felt ready to take the role of a MTE (see ***C*** in Figure 8).

Rachel's growth and transition from MT to MTE was similar to that of Tami – from MTE to MTEE. Thus, Rachel's planning stage of a new workshop involving other sorting tasks (which was done in collaboration with Hanna who underwent the same process) was similar to Tami's initial planning stage (see ***D*** in Figure 8 in comparison to ***a*** in Figure 7). The need Rachel felt to consult with Tami regarding the learning offers she had planned, was somewhat like the need Tami felt to consult with Keren before the first workshop (see ***E*** in Figure 8 in comparison to ***b*** in Figure 7). Again, the exchange of ideas and consultation led to an improved plan of the learning offers that Rachel facilitated for the MTs (see ***F*** in Figure 8 in comparison to ***c*** in Figure 7). Altogether, similar to the way Tami's MTE's knowledge grew through her practice and evolved into a MTEE's knowledge, Rachel's MT's knowledge developed into a MTE's knowledge.

Our analysis conveys the iterative nature of the growth through practice of the different members of the community of mathematics educators and through their different interactions, continuously switching roles from learners to facilitators. The story that we chose to present is one of many similar stories that emerged throughout this five-year project.

CONCLUDING REMARKS

In this paper we introduced a three-layer model of development of members of a community of mathematics educators (Figure 2). The model is used to analyze the growth through practice of teacher educators. By applying it to one particular professional development program, this model is shown to have a descriptive and explanatory power.

Throughout the paper we examine the mathematics educators involved in the project as members of a special community of practice that evolved over time in the context of the project. Our analysis focused on the different opportunities that were offered within this context and the ways in which these opportunities led to the development of the various members of this community, with an emphasis on two different members. In particular, we described interactions between experienced members of the community and new comers who were less experienced. Our analysis is a detailed account of how the "knowledgeability comes from participating in a community's ongoing practices" (Roth, 1998, p. 12). Moreover, it points to the ways in which these interactions contributed not only to the newcomers (e.g., Rachel) but also to the more senior and experienced members of the community (e.g., Tami).

We speculate that the three-layer model that we offer and use in this paper may be useful in similar ways in shedding light on and describing such processes within other professional development frameworks. We also are rather confident that awareness of the complexities involved in MTEs work, as portrayed through the model (see Figure 2), may draw attention to and shed light on the necessary ingredients of this evolving profession. We suggest further that this model may be integrated as part of MTEs education. Thus, fostering MTEs reflection by explicitly referring to the above model and its relevance to their profession may enhance MTEs growth-through-practice.

To conclude, we point to the role that mathematical challenge and the specific mathematical tasks played in this context. A major concern of the mathematics education community has to do with teachers' mathematical knowledge. There is an agreement that teachers must have a deep and broad understanding of school mathematics in order to be able to offer students challenging mathematics. Unfortunately, formal higher education in mathematics does not always address this need. In our work, we have developed numerous activities aimed at this goal (e.g., Zaslavsky, 1995, in press; Leikin, 2003). Our challenge was to identify mathematical subtleties in the junior and senior high school curriculum and to design learning activities that, under certain modifications, are equally applicable to students as well as to MTs. The sorting task, which constituted the core content of the story we presented earlier, is one example of this approach. While such learning activities are the main goal for student mathematical learning, for MTs they served as a vehicle for professional growth beyond the mathematical knowledge.

As indicated in our story, the sorting assignment proved challenging for MTs as well as for MTEs in terms of the mathematics involved. However, it also provided a rich opportunity for teachers to experience a different kind of learning (compared with their previous experiences as learners), which required activeness and cooperation. Reflection on their work as learners was a key factor in the transition from dealing with the mathematics itself to elevating their experience to encompass the MTs teaching triad. Our work may be viewed as an attempt to shift from looking at Jaworski's teaching triad mainly as three distinct interrelated components to treating it as an entity in itself that forms the challenging content of MTEs (See Figure 1).

This process, in itself, of designing such mathematical activities for MTs proved instrumental in the professional growth of MTEs (Zaslavsky, Chapman & Leikin, 2003, refer to this process as an example of indirect learning). The design of such activities required drawing on sound mathematical knowledge, sensitivity to MTs as learners, and implementation of innovative approaches to management of MTs learning. In addition, it also involved a reflective state of mind and a collaborative disposition.

NOTES

[1] This paper shares the same setting and method described in Zaslavsky and Leikin (1999), though the conceptual framework has been refined and extended with reference to a different story.

[2] The mathematics component of the "Tomorrow 98" project in the Upper Galilee was directed by the Technion – Israel Institute of Technology, and funded by the Israeli Ministry of Education.

[3] All the names in the paper are pseudonyms.

REFERENCES

Borasi, R. (1999). Beginning the process of rethinking mathematics instruction: A professional development program. *Journal of Mathematics Teacher Education*, 2(1), 49–78.

Brown, C.A. & Borko, H. (1992). Becoming a mathematics teacher. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209–239). New York: Macmillan.

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the highschool algebra classroom*. New York: Teachers College Press.

Cooney, T.J. (1994). Teacher education as an exercise in adaptation. In D.B. Aichele & A.F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 9–22). Reston, VA: National Council of Teachers of Mathematics.

Cooney, T.J. & Krainer, K. (1996). Inservice mathematics teacher education: The importance of listening. In A.J. Bishop et al. (Eds.), *International handbook of mathematics education* (pp. 1155–1185). The Netherlands: Kluwer Academic Publishers.

Dewey, J. (1933). *How we think: A statement of the relation of reflective thinking to the educative process*. Boston: D.C. Heath and Co.

Even, R. (1999). The development of teacher leaders and inservice teacher educators. *Journal of Mathematics Teacher Education*, 2(1), 3–24.

Erickson, F. (1986). Qualitative methods in research on teaching. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 119–161). New York: Macmillan.

Glaser, B.G. & Strauss, A.L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Howthorne, NY: Aldine.

Jaworski, B. (1992). Mathematics teaching: What is it? *For the Learning of Mathematics*, 12(1), 8–14.

Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: The Falmer Press.

Jaworski, B. (1998). Mathematics teacher research: Process, practice, and the development of teaching. *Journal of Mathematics Teacher Education*, 1(1), 3–31.

Krainer, K. (1998). Some considerations on problems and perspectives of inservice mathematics teacher education. In C. Alsina, J.M. Alvarez, B. Hodgson, C. Laborde & A. Pérez (Eds.), *8th International Congress on Mathematics Education: Selected lectures* (pp. 303–321). S. A. E. M. Thales: Sevilla, Spain.

Krainer, K. (1999). Promoting reflection and networking as an intervention strategy in professional development programs for mathematics teachers and mathematics teacher educators. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education, v.1* (pp. 159–168).

Krainer, K. (2001). Teachers' growth is more than the growth of individual teachers: The case of Gisela. In F.-L. Lin & T.J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 271–293). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven & London: Yale University Press.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, Culture & Activity*, 3(3), 149–164.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Leikin, R. & Zaslavsky, O. (1999). Connecting research to teaching: Cooperative learning in mathematics. *Mathematics Teacher*, 92(3), 240–246.

Leikin, R., Berman, A. & Zaslavsky, O. (2000). Learning through teaching: The case of symmetry. *Mathematics Education Research Journal*, 12, 16–34.

Leikin, R. (2003). Problem-solving preferences of mathematics teachers. *Journal of Mathematics Teacher Education*, 6(4), 297–328.

Mason, J. (1998). Enabling teachers to be real teachers: Necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education*, 1(3), 243–267.

National Council of Teachers of Mathematics (NCTM) (Ed.) (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Noddings, N. (1992). Professionalization and mathematics teaching. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 197–208). New York: Macmillan.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.

Roth, W.-M. (1998). *Designing communities.* Boston: Kluwer Academic Publication.

Schifter, D. (Ed.) (1996). *What's happening in math class? Envisioning new practices through teacher narratives*. Volumes one and two. New York: Teacher College Press.

Schön, D.A. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Silver, E.A. (1979). Student perceptions of relatedness among mathematical verbal problems. *Journal for Research in Mathematics Education*, 10, 195–210.

Steinbring, H. (1998). Elements of epistemological knowledge for mathematics teachers. *Journal of Mathematics Teacher Education*, 1(2), 157–189.

Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, 4, 259–283.

Zaslavsky, O. (1995). Open-ended tasks as a trigger for mathematics teachers' professional development. *For the Learning of Mathematics*, 15(3), 15–20.

Zaslavsky, O. (in press). Seizing the opportunity to create uncertainty in learning mathematics. *Educational Studies in Mathematics*.

Zaslavsky, O., Chapman, O. & Leikin, R. (2003). Professional development in mathematics education: Trends and tasks. In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick & F.K.S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 875–915). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Zaslavsky, O. & Leikin, R. (1999). Interweaving the training of mathematics teacher-educators and the professional development of mathematics teachers. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education, v.1* (pp. 143–158).

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