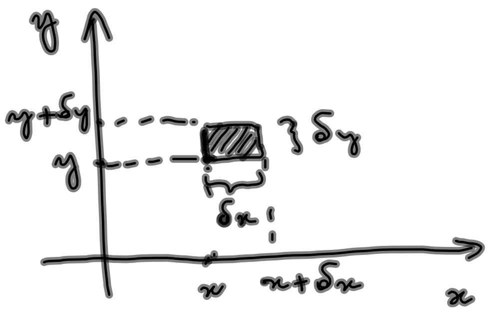
Έννοια 2-διαστ. σππ



$$f_{X,Y}(x,y)$$

$$= \lim_{\substack{\delta x \to 0^+ \\ \delta y \to 0^+}} \frac{P(x \le X \le x+\delta x,\ y \le Y \le y+\delta y)}{\delta x\, \delta y}$$

Αναλογία διακριτών-συνεχών
(πολυδιάστατων)

Διακρ.	Συνεχής
$P_{X,Y}(x,y)$ σπ ή σμπ	$f_{X,Y}(x,y)$ σππ
$= P(X=x, Y=y)$	$= \lim_{\substack{\delta x \to 0^+ \\ \delta y \to 0^+}} \frac{P(x\le X < x+\delta x,\ y \le Y \le y+\delta y)}{\delta x \, \delta y}$
Περιθ. σπ	Περιθ σππ
$P_X(x) = \sum_y P_{X,Y}(x,y)$	$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy$

$$P(a \le X \le b,\ c \le Y \le d) = \sum_{x=a}^{b} \sum_{y=c}^{d} P_{X,Y}(x,y)$$

$$P(a \le X \le b,\ c \le Y \le d) = \int_a^b \int_c^d f_{X,Y}(x,y)\,dy\,dx$$

$$E(g(X,Y)) = \sum_x \sum_y g(x,y)\, P_{X,Y}(x,y)$$

$$E(g(X,Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\, f_{X,Y}(x,y)\,dx\,dy$$

$$E(aX + bY + c) = aE(X) + bE(Y) + c$$

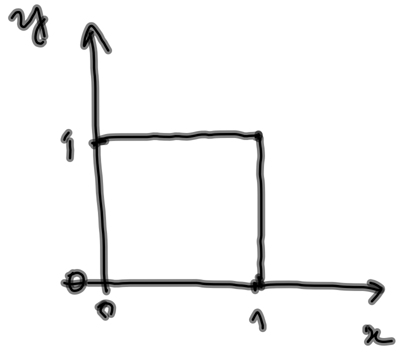
$$Var(X) = E(X^2) - (E(X))^2$$

$$Var(aX+b) = a^2\, Var(X)$$

Δεκ 9-9:31 πμ

Άσκηση 1: ΣΠΠ σε ορθογώνιο

(X, Y) 2-διάστ. τ.δ. συν. με τιμές στο $[0,1]^2$



$$f_{X,Y}(x,y)=\begin{cases} cxy^2, & 0\le x,y\le 1 \\ 0, & \text{διαφ.}\end{cases}$$

$$c=6$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy=1$$

$$\Rightarrow \int_0^1\int_0^1 cxy^2\,dx\,dy=1 \Rightarrow c\int_0^1 y^2\int_0^1 x\,dx\,dy=1$$

$$\Rightarrow c\int_0^1 y^2\,\frac{x^2}{2}\Big|_{x=0}^{1}dy=1 \Rightarrow \frac{c}{2}\int_0^1 y^2\,dy=1 \Rightarrow \frac{c}{2}\,\frac{y^3}{3}\Big|_{y=0}^{1}=1 \Rightarrow$$

$$f_{X,Y}(x,y) = \begin{cases} 6xy^2 & , 0 \le x \le 1 , 0 \le y \le 1 \\ 0 & , \delta\iota\alpha\varphi. \end{cases}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy =$$

$$= \begin{cases} \int_0^1 6xy^2 dy & 0 \le x \le 1 \\ 0, & x<0 \text{ ή } x>1 \end{cases}$$

$$= \begin{cases} 6x \int_0^1 y^2 dy = 6x \frac{y^3}{3}\Big|_{y=0}^{1} = 2x , & 0 \le x \le 1 \\ 0, \ \delta\iota\alpha\varphi. \end{cases}$$

Όμοια:

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \delta\iota\alpha\varphi. \end{cases} \qquad f_Y(y) = \begin{cases} 3y^2 , & 0 \le y \le 1 \\ 0 , & \delta\iota\alpha\varphi. \end{cases}$$

$$P\left(X > \frac{1}{2},\ \frac{1}{3} \le Y \le \frac{2}{3} \right)$$

$$= \int_{1/2}^{1} \int_{1/3}^{2/3} 6xy^2 \, dy \, dx$$

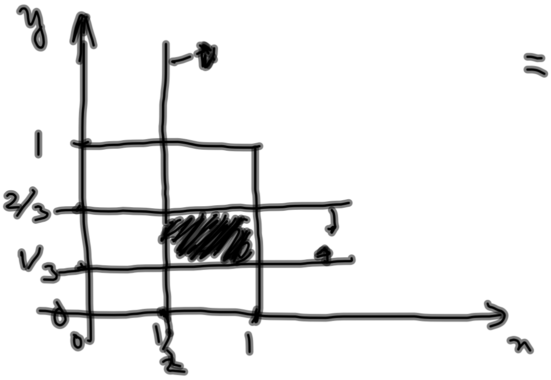
$$= \dots$$

$$E(\underbrace{XY}_{g(X,Y)}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f_{X,Y}(x,y)\, dx\, dy$$

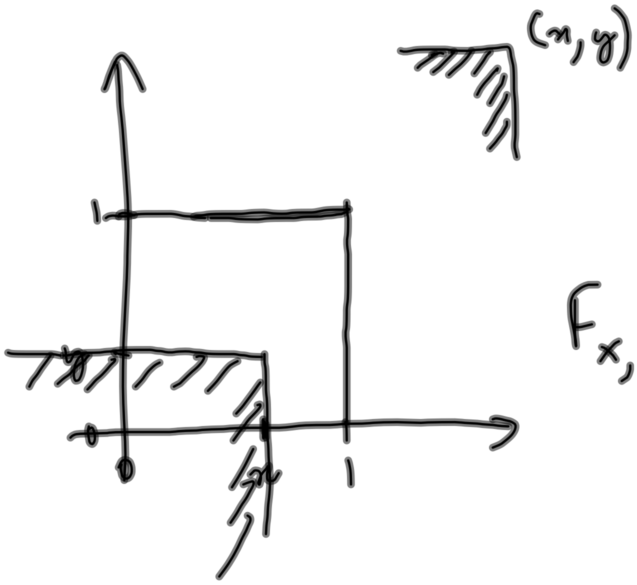
$$= \int_0^1 \int_0^1 xy \, 6xy^2 \, dy \, dx$$

$$= \int_0^1 \int_0^1 6x^2y^3 \, dy \, dx$$

$$= \dots$$

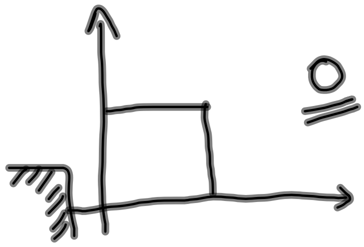


$$F_{X,Y}(x,y) = P(X \le x,\ Y \le y)$$



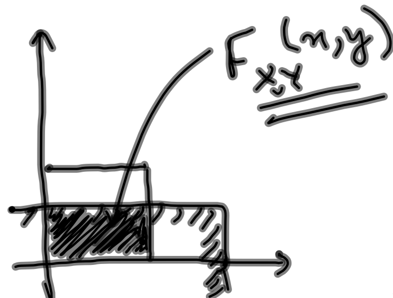
$$F_{X,Y}(x,y) = \begin{cases} 0, & x<0 \text{ ή } y<0 \\ \int_0^y f_Y(t)\,dt & x>1,\ 0\le y\le 1 \\ \int_0^x f_X(s)\,ds & y>1,\ 0\le x\le 1 \\ \int_0^x\int_0^y f_{X,Y}(s,t)\,dt\,ds & 0\le x\le 1,\ 0\le y\le 1 \\ 1 & x>1,\ y>1 \end{cases}$$

$F_{X,Y}(x,y)$

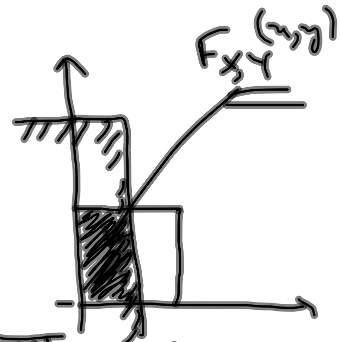


$\underline{\underline{0}}$

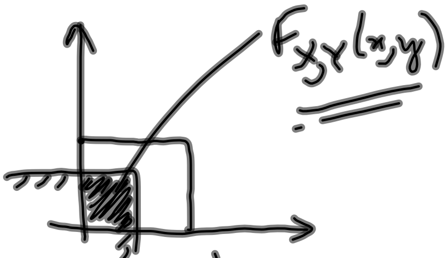
I: $x<0$ ή $y<0$



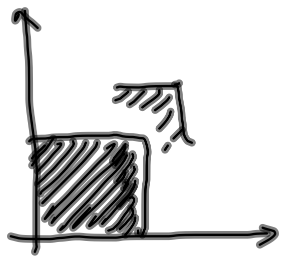
II: $x\geq 1$, $0\leq y\leq 1$



III: $y>1$, $0\leq x\leq 1$



IV: $0\leq x\leq 1$
$0\leq y\leq 1$

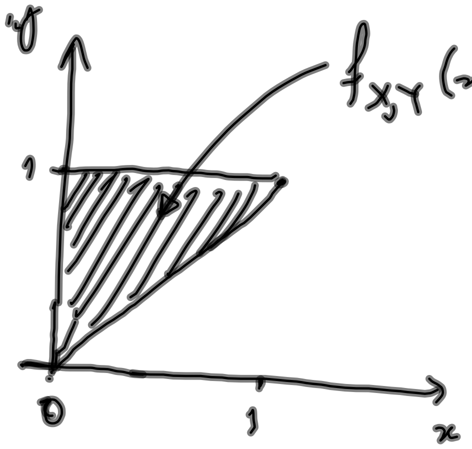


$\underline{\underline{1}}$

V: $x\geq 1$, $y\geq 1$

Άσκηση 2: Σππ σε τρίγωνο

$$f_{X,Y}(x,y) = \begin{cases} c\,x y^2, & 0 \le x \le y \le 1 \\ 0, & \text{διαφ.} \end{cases}$$



$f_{X,Y}(x,y) > 0$

$$0 \le x \le y \le 1$$
$$\Updownarrow$$
$$\{0 \le x \le 1, \quad x \le y \le 1\}$$
$$\Updownarrow$$
$$\{0 \le y \le 1, \quad 0 \le x \le y\}$$

$c = ;$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy = 1$$

$$\boxed{0 \le x \le y \le 1}$$

$$\Rightarrow \int_0^1 \int_0^y c\, x y^2 \quad dx\, dy = 1 \leftarrow \begin{array}{l} 0 \le y \le 1 \\ 0 \le x \le y \end{array}$$

$$\text{ή} \quad \int_0^1 \int_x^1 c\, x y^2 \quad dy\, dx = 1 \leftarrow \begin{array}{l} 0 \le x \le 1 \\ x \le y \le 1 \end{array}$$

$$\Rightarrow c \int_0^1 y^2 \int_0^y x\, dx\, dy = 1 \Rightarrow c \int_0^1 y^2 \left. \frac{x^2}{2} \right|_{x=0}^{y} dy = 1$$

$$\Rightarrow c \int_0^1 \frac{y^4}{2}\, dy = c \left. \frac{y^5}{10} \right|_{y=0}^{1} = \frac{c}{10} = 1 \Rightarrow c = 10$$

$$f_{X,Y}(x,y) = \begin{cases} 10xy^2, & 0 \le x \le y \le 1 \\ 0, & \delta\iota\alpha\varphi. \end{cases}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$= \begin{cases} 0, & x < 0 \text{ ή } x > 1 \\ \int_x^1 10xy^2 dy = 10x \int_x^1 y^2 dy = 10x \dfrac{1-x^3}{3} & 0 \le x \le 1 \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

$$= \begin{cases} 0, & y<0 \text{ ή } y>1 \\ \int_0^y 10xy^2\,dx = 10y^2\int_0^y x\,dx = 5y^4, & 0 \le y \le 1 \end{cases}$$

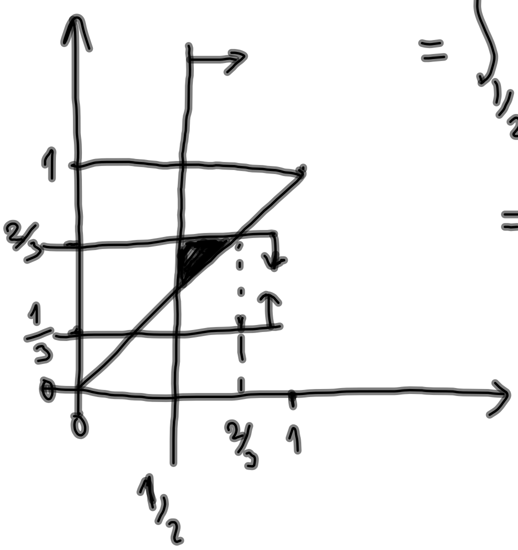
Δεκ 9-10:38 πμ

$$P\left(X > \frac{1}{2},\ \frac{1}{3} \le Y \le \frac{2}{3}\right) =$$

$$= \int_{1/2}^{\infty} \int_{1/3}^{2/3} f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_{1/2}^{2/3} \int_{x}^{2/3} 10\, x y^2 \; dy\, dx$$

$$= \dots$$



$0 \le x \le y \le 1$

$x > \frac{1}{2},\ \frac{1}{3} \le y \le \frac{2}{3}$

$\boxed{\frac{1}{2} \le x \le \frac{2}{3} \ / \ x \le y \le \frac{2}{3}}$

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy\, f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_{0}^{1} \int_{x}^{1} xy\, 10xy^2\, dy\, dx$$

$$= \ldots$$

Παρ: Χρόνος στη στάση

X = Χρόνος άφιξης , Y = Χρόνος αναμ.

A = { φθάνω μεταξύ 7:10 με 7:15 }

A^c = { φθάνω 7:15 7:30 }

$$f_Y(y) = P(A) f_{Y|A}(y) + P(A^c) f_{Y|A^c}(y)$$

$$= \frac{1}{4} f_{Y|A}(y) + \frac{3}{4} f_{Y|A^c}(y)$$

$$= \frac{1}{4} f_{Y|A}(y) + \frac{3}{4} f_{Y|A^c}(y)$$

$$= \cdots$$

Είναι όμως

$$f_{Y|A}(y) = \begin{cases} \frac{1}{5}, & 0 \le y \le 5 \\ 0, & \text{διαφ.} \end{cases}$$

$$f_{Y|A^c}(y) = \begin{cases} \frac{1}{15}, & 0 \le y \le 15 \\ 0, & \text{διαφ.} \end{cases}$$

οπότε

$$f_Y(y) = \begin{cases} \frac{1}{4} \cdot \frac{1}{5} + \frac{3}{4} \cdot \frac{1}{15} = \frac{1}{10}, & 0 \le y \le 5 \\ \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot \frac{1}{15} = \frac{1}{20}, & 5 < y \le 15 \\ 0, & \text{διαφορετικά} \end{cases}$$