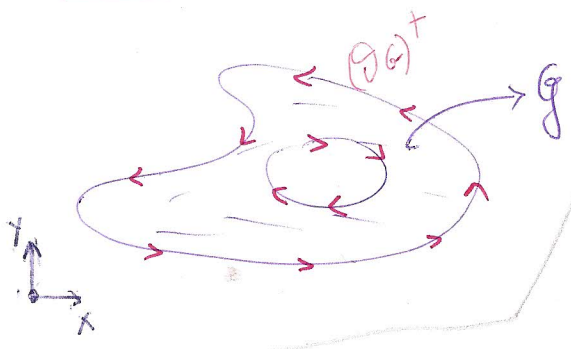


Εφ. Μορφή Θ. Green

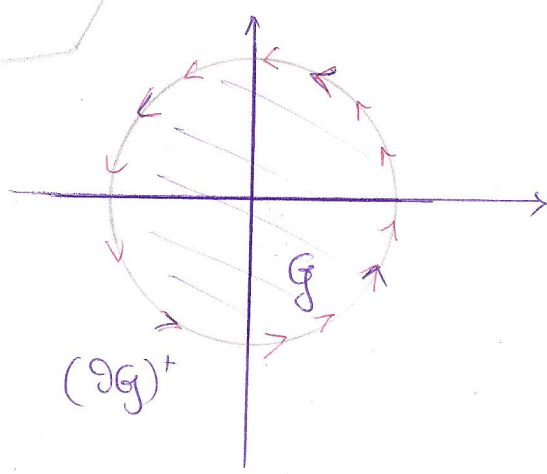
$G$  = εστοιχειώδης σύνολο Green,  $(\partial G)^+$  = σύνορο του  $G$  θετικά προσανατολισμένα.

$$\int_{(\partial G)^+} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt = \iint_G \left( \frac{\partial Q}{\partial x}(x,y) - \frac{\partial P}{\partial y}(x,y) \right) dx dy \quad (I)$$



Άσκησης

1) Να επαληθευτεί ο τύπος (I) για  $\vec{F}(x,y) = (x+y, y)$  στο  $G = \{(x,y) : x^2 + y^2 \leq 1\}$  (το  $G$  είναι αθρόο σύνολο Green)



$$\begin{aligned} (\partial G)^+ : \vec{r}(t) &= (\cos t, \sin t), \quad t \in [0, 2\pi] \\ \vec{r}'(t) &= (-\sin t, \cos t) \\ \vec{F}(\vec{r}(t)) &= \vec{F}(\cos t, \sin t) = (\sin t + \cos t, \sin t) \end{aligned}$$

$$\begin{aligned} I_1 &= \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt = \int_0^{2\pi} (\sin t + \cos t, \sin t) \cdot (-\sin t, \cos t) dt = \\ &= \int_0^{2\pi} (-\sin^2 t - \sin t \cos t + \sin t \cos t) dt = - \int_0^{2\pi} \sin^2 t dt = -\pi \end{aligned}$$

$I_1 = -\pi$



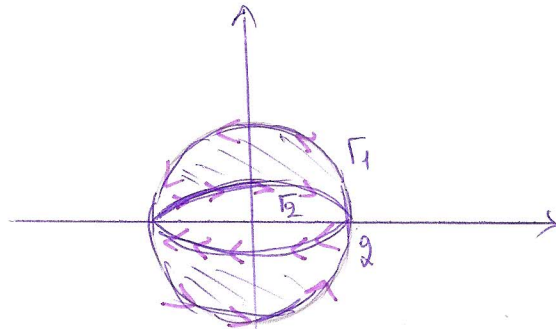
$$\vec{F} = (P, Q), \quad P(x,y) = x+y, \quad Q(x,y) = y$$

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$$I_2 = \iint_G \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \iint_G (0-1) dx dy = - \iint_G dx dy = -\pi$$

$$\boxed{I_2 = -\pi} \quad \underline{\text{Έτσι}} \quad I_1 = I_2$$

2) Να επαληθεύσει ο ένος (I) για  $\vec{F}(x,y) = (4x-2y, 2x+6y)$   
 στο  $G = \left\{ (x,y) : x^2+y^2 \leq 4, \left(\frac{x}{2}\right)^2 + y^2 \geq 1 \right\}$  (ακριβώς όπως είναι)



$$I_1 = \int_{\partial G^+} (\vec{F} \cdot \vec{n}) dt$$

$$\vec{\Gamma}_1^+ : \vec{r}_1(t) = (2\cos t, 2\sin t), \quad t \in [0, 2\pi] \quad [x^2 + y^2 = 4]$$

$$\vec{\Gamma}_2^+ : \vec{r}_2(t) = (\cos t, \sin t), \quad t \in [0, 2\pi] \quad \left[\left(\frac{x}{2}\right)^2 + y^2 = 1\right]$$



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$$\bullet I_1 = \int_{(\partial G)^+} (\vec{F} \cdot \vec{r}') dt = \int_0^{2\pi} \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1'(t) dt - \int_0^{2\pi} \vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2'(t) dt =$$

$$\int_0^{2\pi} (86wt - 4nkt, 46wt + 12nkt) \cdot (-2nkt, 26wt) dt -$$

$$- \int_0^{2\pi} (8wt - 2nkt, 46wt + 6nkt) \cdot (-2nt, 6wt) dt = \dots$$

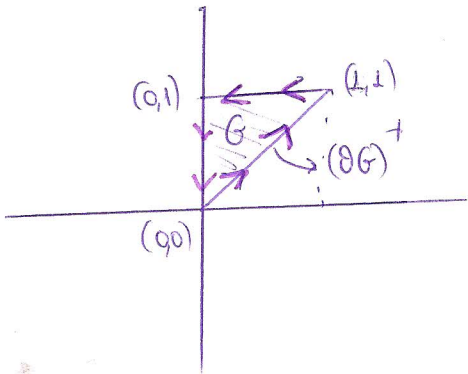
$$= \int_0^{2\pi} (4 + 18nkt^2) dt = \dots = 8\pi$$

$$I_2 = \iint_G \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \iint_G (2 - (-2)) dx dy = 4 \iint_G dx dy =$$

$$= 4(\pi \cdot 2^2 - \pi \cdot 2) = 8\pi$$

$$\left( \begin{aligned} \text{Εμβαδόν } G &= (\text{Εμβαδόν κύκλου ακτίνας } a=2) - (\text{Εμβαδόν έλλειψος με } a=2, b=1) = \\ &= (\pi a^2)_{a=2} - (\pi a b)_{a=2, b=1} = \pi 2^2 - \pi \cdot 2 = 2\pi \end{aligned} \right)$$

3) Να υπολογιστεί η ποση του  $\vec{F}(x,y) = (xy-x^2, x^2y)$  κατά μήκος του εμβαδού του τριγώνου  $G$  με κορυφές τα  $(0,0), (0,1), (1,1)$  -226-



$$G = \{(x,y) : 0 \leq x \leq 1, x \leq y \leq 1\} \quad (x-0) \hat{i} \hat{j}$$

$$= \{(x,y) : 0 \leq y \leq 1, 0 \leq x \leq y\} \quad (y-x) \hat{j}$$

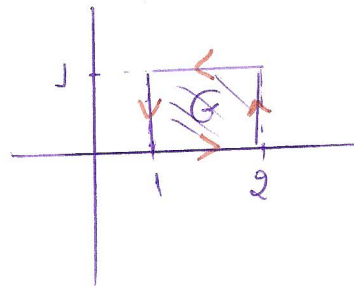
(από τον Τόμο (I) 64.223)

$$I = \iint_G \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \int_0^1 \left( \int_x^1 (2xy - x) dy \right) dx =$$

$$= \int_0^1 (xy^2 - xy) \Big|_x^1 dx = - \int_0^1 (y^3 - y^2) dy =$$

$$= -\frac{1}{4} + \frac{1}{3} = \frac{1}{12} //$$

4) Να υπολογιστεί η ποση του  $\vec{F}(x,y) = (x-xy, y^3+1)$  κατά μήκος του εμβαδού του  $G = [1,2] \times [0,1]$



$$I = \int_1^2 \left( \int_0^1 (0 - (-x)) dy \right) dx =$$

$$= \int_1^2 x dx = \frac{x^2}{2} \Big|_1^2 = \frac{3}{2} //$$