Ανάλυση ΙΙ – 14/03/2011 – μάθημα 08

Εφαρμογή των μετασχηματισμών του αρχείου Α)

Παραδείγματα στον $\mathbb{R}^2$

(A) Να μετατραπούν σε πολικές εξισώσεις οι εξισώσεις των καμπυλών:

1) $\Gamma_1 : x^2 + y^2 = 16$

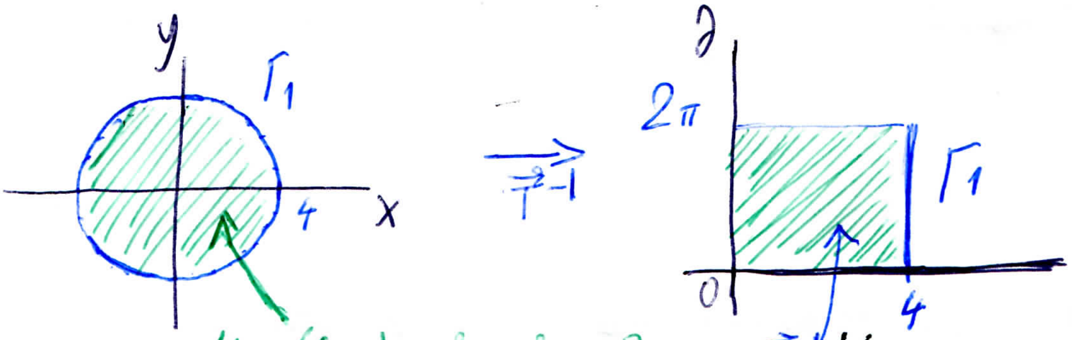
Έχουμε:

$$\left.\begin{aligned} x &= r\,\sigma\upsilon\nu\theta \\ y &= r\,\eta\mu\theta \end{aligned}\right\}, \quad r = \sqrt{x^2+y^2} = 4$$

$\Gamma_1 : r = 4,\ \theta \in [0, 2\pi]$

Παραμ.: $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta), \theta$

$\cdot\ x^2 + y^2 = 16$ άρα $r = 4$, $\theta \in [0, 2\pi]$



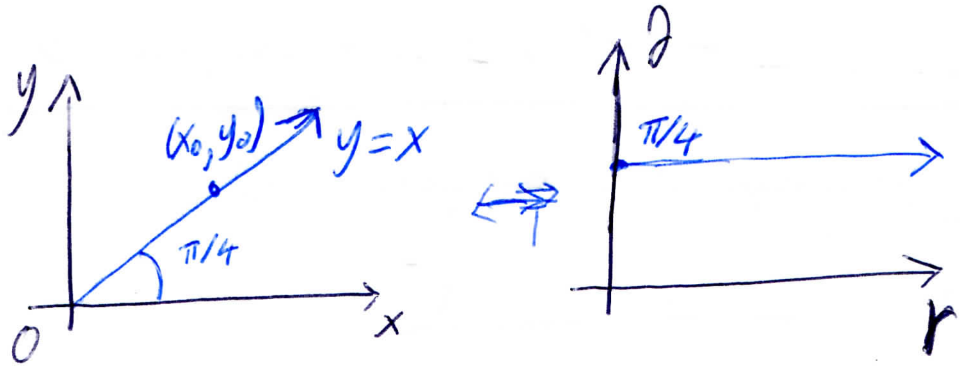
$K = \{(x,y) : x^2 + y^2 \leq 16\}$ $\quad \vec{T}^{-1}(K) = \{(r,\theta) : 0 \leq r \leq 4,\ \theta \in [0,$

2) $y = x$, $x \geq 0$

$\varepsilon\varphi\theta = 1$ $\quad \left(\begin{aligned} x &= r\,\sigma\upsilon\nu\theta \\ y &= r\,\eta\mu\theta \end{aligned}\right)$

$\theta = \frac{\pi}{4}$, $r \geq 0$

Παρ. $\vec{r}(r) = \{(r\frac{1}{\sqrt{2}}, r\frac{1}{\sqrt{2}}), r \geq 0\}$

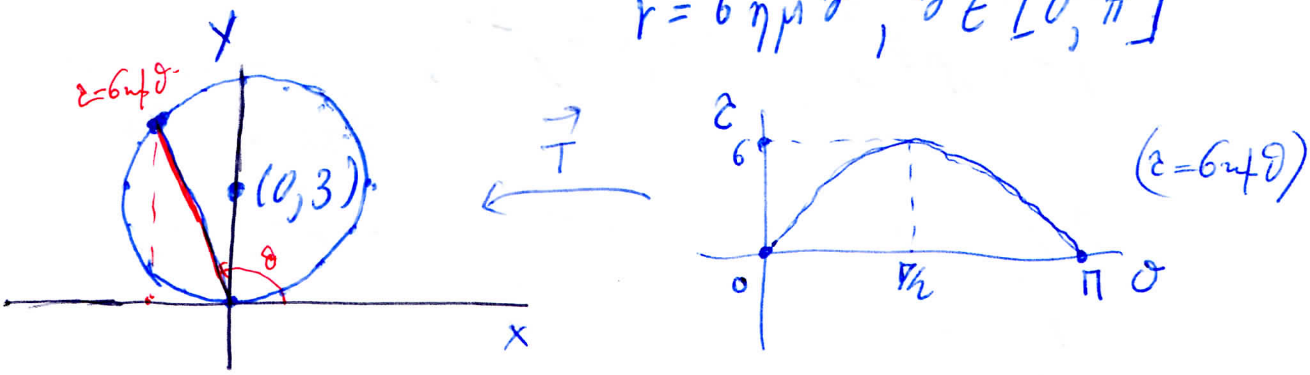


3) $x^2+y^2-6y=0$

$(x^2+(y-3)^2=9)$

$r^2-6r\,\eta\mu\theta=0$

$r=6\,\eta\mu\theta\,,\ \theta\in[0,\pi]$



Ⓑ Να μετατραπούν σε καρτ. εξισώσεις (x,y) οι πολικές εξισώσεις των καμπυλών:

1) $r=1\,,\quad \theta\in[0,\pi/2]$ (Το r δεν εξαρτάται από την γωνία

$\begin{Bmatrix} x^2+y^2=1 \\ x,y\geq 0 \end{Bmatrix}$ $\quad \begin{pmatrix} x=\sigma\upsilon\nu\theta \\ y=\eta\mu\theta \end{pmatrix}$ $\quad \theta\in[0,\frac{\pi}{2}]$

2) $r = 4\,\sigma\upsilon\nu\theta \qquad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(Το r εξαρτάται από το θ)

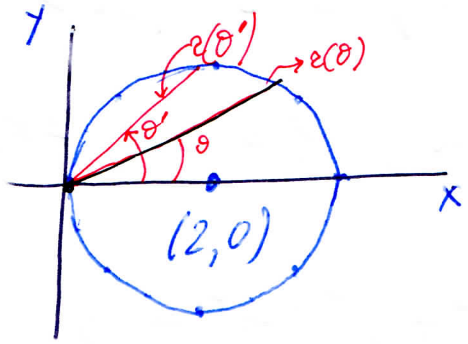
$r^2 = 4r\,\sigma\upsilon\nu\theta$

ή

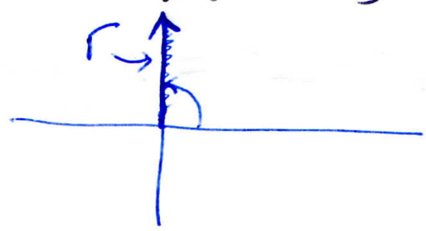
$\theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

$x^2 + y^2 = 4x$

$(x-2)^2 + y^2 = 4$



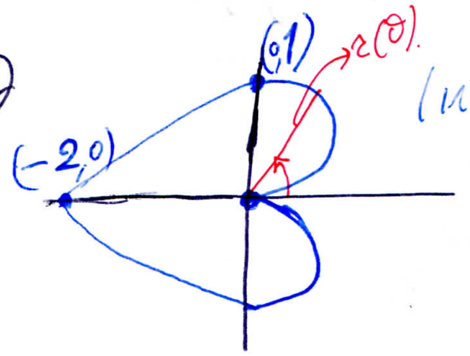
3) $\theta = \frac{\pi}{2}, \quad \{(x,y) : x = 0,\ y \geq 0\} = \Gamma$



$x = r\,\sigma\upsilon\nu\theta = 0$

$y = r\,\eta\mu\theta = r \geq 0$

4) $r = 1 - \sigma\upsilon\nu\theta$



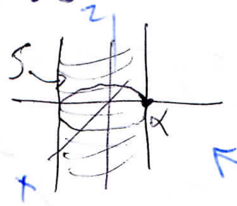
(καρδιοειδής)

$r^2 = r - r\,\sigma\upsilon\nu\theta$

$x^2 + y^2 = \sqrt{x^2+y^2} - x, \quad (x^2+y^2+x)^2 = x^2 + y^2$

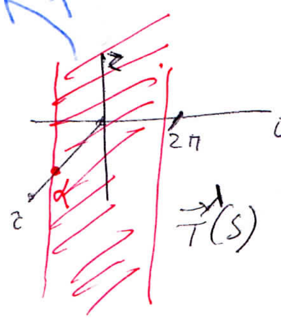
Παραδείγματα στον $\mathbb{R}^3$

Ⓐ Καρτεσιανές → Κυλινδρικές



1) $x^2+y^2=a^2 \quad (a>0)$

$r=a, \quad \theta \in [0, 2\pi] \quad z \in \mathbb{R},$

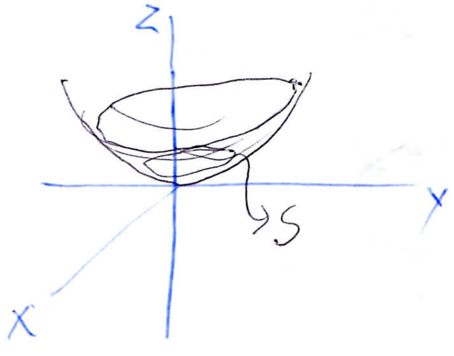


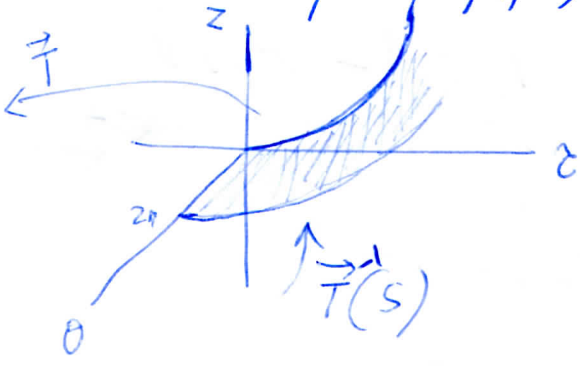
2) $x^2+y^2+z^2=y$

$x^2+\left(y-\frac{1}{2}\right)^2+z^2=\frac{1}{4}$

$r^2+z^2=r\,\eta\mu\theta$

3) $z=x^2+y^2, z=r^2$ (παραβολοειδές εκ περιστροφής)

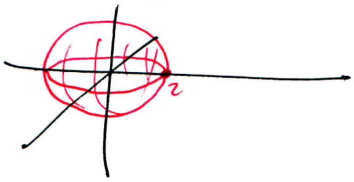




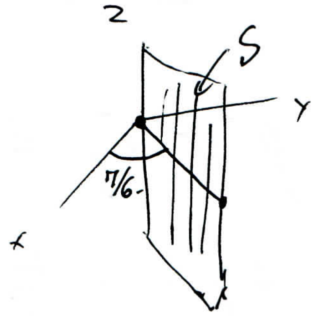
(B) Να μετατραπούν οι εξισώσεις από Κυλινδρικές → Καρτεσιανές.

1) $r^2 + z^2 = 4$

$x^2 + y^2 + z^2 = 4$



2) $\theta = \frac{\pi}{6}$, $x = r\,\sigma\upsilon\nu\theta$, $y = r\,\eta\mu\theta$



$\frac{y}{x} = \varepsilon\varphi\,\theta = \frac{1}{\sqrt{3}}$

$S = \{(x, y, z) \in \mathbb{R}^3 : y = \frac{1}{\sqrt{3}} x,\ x \geq 0,\ z \in \mathbb{R}\}$

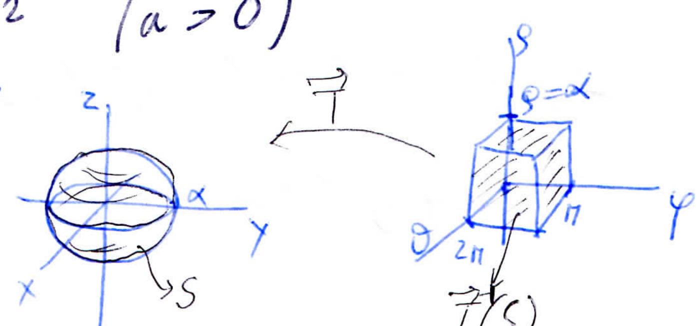
3) $r = 9$, $S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 81,\ z \in \mathbb{R}\}$ (κύλινδρος)

(Γ) Καρτεσιανές → Σφαιρικές

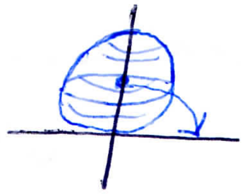
1) $x^2 + y^2 + z^2 = a^2$ $(a > 0)$

$\rho = a$, $\theta \in [0, 2\pi]$

$\varphi \in [0, \pi]$



2) $x^2+y^2+z^2=z$



$\rho^2=\rho$ συνφ

$\rho=$ συνφ, $\theta\in[0,2\pi]$, $\varphi\in[0,\frac{\pi}{2}]$

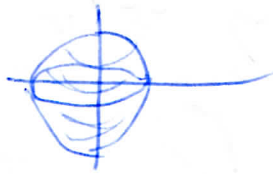
3) $z^2=2x^2+2y^2,\ x,y\geq 0$

ρ^2συν$^2\varphi=2\rho^2$ημ$^2\varphi$

εφφ $=\frac{1}{\sqrt{2}}$, $\varphi=$ τοξεφ $\frac{1}{\sqrt{2}}$ $(\theta\in[0,\frac{\pi}{2}],\ \rho\geq 0)$

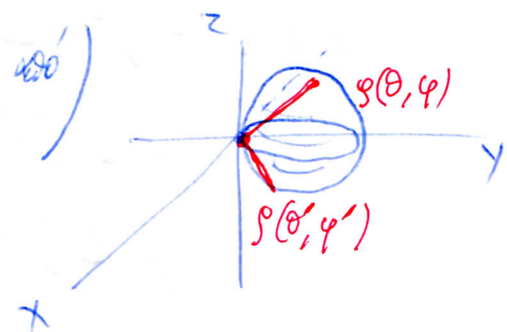
Δ Σφαιρικές → Καρτεσιανές

1) $\rho=10$, $x^2+y^2+z^2=10^2$
(Δεν εξαρτάται από τα θ,φ) (Σφαίρα)



2) $\rho=$ ημθ ημφ (Εξαρτάται από τα θ,φ)

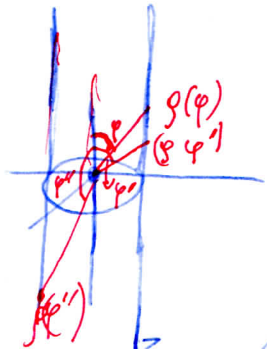
$x^2+y^2+z^2=y$



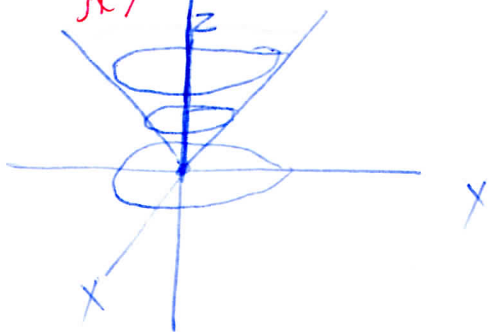
3) $\rho\ \eta\mu\varphi = 10$

$$x^2+y^2=\rho^2\,\eta\mu^2\varphi$$

Άρα $x^2+y^2=10^2$



4) $\varphi = \frac{\pi}{4}$ (κώνος)



$$x=\rho\ \sigma\upsilon\nu\theta \cdot \eta\mu\frac{\pi}{4}$$

$$y=\rho\,\eta\mu\theta\ \ \eta\mu\frac{\pi}{4}$$

$$z=\rho\,\sigma\upsilon\nu\frac{\pi}{4}\quad (\geq 0)$$

Άρα $\left.\begin{aligned} x^2+y^2&=\rho^2\,\eta\mu^2\frac{\pi}{4} \\ z^2&=\rho^2\,\sigma\upsilon\nu^2\frac{\pi}{4}\end{aligned}\right\}\ \frac{x^2+y^2}{z^2}=1$

$$\Rightarrow z^2=x^2+z^2,\ z\geq 0$$

$\left.\begin{array}{l}\text{Οι πολικές στον } \mathbb{R}^2 \\ \text{Οι κυλινδρικές-σφαιρικές στον } \mathbb{R}^3\end{array}\right\}$ χρειάζονται στα $\left\{\begin{array}{l}\text{Διπλά ολοκ} \\ \text{Τριπλά ολοκλ.} \\ \text{Επικαμπύλια ολ} \\ \text{Επιφανειακά ολ} \\ \text{Θ. Green, Stokes, Gau}\end{array}\right.$

Μαθαίνουμε <u>ΤΕΛΕΙΑ</u> !!!!!!!!!!!!!!!!!!!!!!!!